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A thermodynamic investigation of the centripetal gas turbine and its part load characteristics

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THERMODYNAMIC INVESTIGATION
OF THE CENTRIPETAL GAS TURBINE
AND ITS PART LOAD CHARACTERISTICS.

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The authors wish to acknowledge with profound appreciation, the suggestion of this thesis subject by Professor Neil P. Bailey, Head of the Mechanical Engineering Department of Rensselaer Polytechnic Institute. Professor Bailey's untiring interest and encouragement was a continuous source of inspiration to the authors.

ABSTRACT OF CONCLUSIONS

The centripetal gas turbine has the inherent characteristic of variable nozzle area. This characteristic tends toward greater cycle stability when changing the fractional load operating conditions.

Short acceleration periods are obtained when in need of quick changes of R.P.M. primarily because of the variable nozzle area effect.

The adiabatic efficiency of the turbine is very high, approximately 90% efficiency has been obtained in actual tests of this type of turbine.

The turbine, when used in a marine gas turbine cycle, proved very flexible. Using the temperature control method, a speed range of 12 to 20 knots can be obtained in a destroyer of the U.S.S. Wadsworth DD60 type. The above speeds can be obtained with efficiencies varying from, 18% at 12 knots to 36% at full load. The temperature control method, when used in a radial type turbine, results in higher part load efficiencies than those obtained in an axial type turbine.

The centripetal turbine is inherently a low pressure turbine, however, this limitation is for only one stage. Thus multistage turbines could be used when higher pressure ratios are desired.

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I

INTRODUCTION

The gas turbine as a prime mover has been slow in coming of age, because its principle elements, the compressor and the turbine, had to be highly perfected before the cycle would produce any net output. To date, extensive research has been conducted on improving the compressor and turbine characteristics, with the result that we have three types of compressors, but only one type of turbine, the axial flow.

Sound reasons for the almost exclusive use of the axial type turbine are non existent. On the contrary, the radial turbine offers certain advantages, which cannot be obtained with the axial type.

To exhaust all possibilities in the field of turbines for gaseous substances, an investigation of the radial type turbine must be included.

The radial turbine proposed in this thesis, is a centripetal type turbine. This turbine, as the name implies, is in principle a centrifugal compressor with its diffuser replaced by nozzles and the flow of gas through the turbine being just the reverse of that in a centrifugal compressor.

Publications concerning theoretical, experimental or practical work with radial turbines of this type are almost unknown. As far as the authors have been able to deduce, only two turbines of this type have been built and tested. One turbine was built in Germany prior to the war and the other by General Electric Company in Schenectady, New York. A theoretical investigation using one dimensional flow theory was made by Dr. Werner T. Von Der Nuell of Germany.[(1)] He investigated design characteristics and overall turbine efficiencies. Thus, it is hoped that this paper on radial type turbines will meet with some interest.

The objectives of this investigation are as follows:

1. To determine what variables of flow are involved in the radial type turbine, and to derive a turbine equation which will determine the relationship between the variables.

2. To analyze the turbine equation as derived and to investigate fully its implied limitations and thermodynamic properties of flow.

3. To determine the effect of the radial turbine characteristics, when used in a marine gas turbine cycle operating at fractional load conditions.

The basic control method of a gas turbine unit is temperature variation. When the gases flow thru a fixed turbine nozzle area, a temperature change produces a flow change, in the opposite direction, which is undesirable. This characteristic of axial turbines can be of such magnitude as to cause instability in the system. The ideal solution to the above undesirable characteristic is to use nozzles with variable area control. The mechanical difficulties involved in variable area nozzles as yet have not been solved.

The centripetal turbine has in effect, the equivalent of a variable nozzle area. The total pressure drop across the turbine is divided between the nozzles and the centrifugal pressure of the impeller. A rise in temperature reduces the gas density in the impeller and consequently reduces the centrifugal pressure. This leaves a greater pressure drop for the nozzles, which tends to increase flow and compensate the effect of the original temperature increase. This characteristic of the centripetal turbine, is very desirable and lends itself ideally toward the basic temperature control method.

This theoretical investigation was undertaken by, Lt. Cdr. A. E. Lundgren, USN, and Lt. W. M. Fowden, USN at Rensselaer Polytechnic Institute, Troy, New York. The investigation was undertaken as partial fulfillment of the requirements for the degree of Master of Science, from January to June 1948.

The authors are greatly indebted to Professor Neil P. Bailey, who suggested this subject and guided the investigation to completion. The authors also wish to express their thanks to General Electric Company for the compressor data used in this thesis.

II

ANALYSIS AND DEVELOPMENT OF PROCEDURE

To make a thermodynamic analysis of the centripetal turbine one must examine its component parts, namely the nozzle and impeller. The characteristics of flow thru a nozzle are well established, but the conditions of flow thru an impeller are more complex. For this first analysis, the non flow, solid forced vortex impeller theory was used. In deriving the turbine equation, the following assumptions were made:

(a) Non flow, solid forced vortex impeller theory.

(b) Adiabatic impeller compression.

(c) Ideal Nozzle. (Reversible)

(d) Mach number at nozzle exit and impeller inlet were equal.

(e) Frictional effects were neglected.

(f) Nozzles were set tangent to impeller circumference. (Zero nozzle angle)

After the above assumptions had been made, the following turbine equation resulted. (See appendix (B) for complete derivation.)

$$\frac{W\sqrt{T_1}}{A_2\,P_1} = \sqrt{\frac{2kg}{k-1}} \times \sqrt{1 - \left[\frac{P_3}{P_1}^{\frac{k-1}{k}} + \frac{k-1}{2}\,\frac{\omega^2 r^2}{kgRT_1}\right]} \times \left[\frac{P_3}{P_1}^{\frac{k-1}{k}} + \frac{k-1}{2}\,\frac{\omega^2 r^2}{kgRT_1}\right]^{\frac{1}{k-1}}$$

.... Equation (2.1)

In the above equation the following subscripts were used:

Subscript (1) Nozzle entrance conditions.

Subscript (2) Nozzle exit conditions.

Subscript (3) Turbine discharge conditions.

The above equation relates weight flow per second per unit nozzle area, $\left(\frac{W}{A_2}\right)$; pressure drop thru the turbine, $\left(\frac{P_1}{P_3}\right)$; total temperature at nozzle entrance (T_1) and the tip speed of the impeller (ωr).

The turbine equation (2.1) resulted in a rather complex form and contained four dependent variables. To simplify the solution of the equation, it was necessary to rearrange the variables into parameters, each containing two variables, one of which would be common to all parameters. The expression $\left(\frac{r^2\omega^2}{kgRT_1}\right)$ was chosen as the main turbine parameter and the following equation resulted upon rearranging the term of equation (2.1). (See appendix (B) for derivation.)

$$\frac{W\omega r}{A_2P_1} = kg\sqrt{\frac{2}{k-1}}\sqrt{\frac{\omega^2r^2}{kgRT_1}}\sqrt{1-\left[\left(\frac{P_3}{P_1}\right)^{\frac{k-1}{k}}+\frac{k-1}{2}\,\frac{\omega^2r^2}{kgRT_1}\right]} \times \left[\left(\frac{P_3}{P_1}\right)^{\frac{k-1}{k}}+\frac{k-1}{2}\,\frac{\omega^2r^2}{kgRT_1}\right]^{\frac{1}{k-1}}$$

...... Equation (2.2)

From equation (2.2) it was evident that a family of curves could be plotted of $\left(\frac{W\omega r}{A_2P_1}\right)$ versus $\left(\frac{\omega^2r^2}{kgRT_1}\right)$ for various values of $\left(\frac{P_3}{P_1}\right)$. (See figure 1)

Using the above curve, a graphical solution to Equation (2.2) resulted when any three of the variables were fixed. This curve, proved to be an expeditious method of solving the turbine equation.

To determine what effects the radial turbine characteristics would have on a typical gas turbine cycle, the authors chose a simple cycle, operating within the limits governed by temperature and wheel tip speed. The cycle selected was the Brayton reheat with regeneration. Maximum temperature at nozzle entrance was set at 1500°F and the maximum wheel tip speed was 1200 feet per second. The cycle investigated was arranged as shown in sketch (one). This cycle arrangement was chosen to allow full use of the temperature control method on turbine number one, maintaining the temperature to turbine number two, at a constant value of 1500°F. Fixing the temperature on the second turbine, eliminated one of the four variables resulting in a unique solution of equation (2.1)

Since the parameter selected in the development of the radial turbine theory is also a common parameter of centrifugal compressor design, it was decided to use this type of compressor. The resulting compressor turbine unit is therefore very simple and compact and should be of extremely light weight per unit power output. Centrifugal compressor data was available for one family of compressors which had been developed by General Electric Company.[2]

Using this data, the authors developed a theoretical double flow, first stage and a single flow, second stage compressor, having the same general characteristics as the original family of compressors. The characteristic curves of this compressor are shown in Figures two, three, and four. For complete development of characteristics, see appendix (C). The governing variable of the compressor was tip speed and for the family of compressors selected the maximum tip speed was determined as 1020 ft. per second. Above this value of tip speed, the adiabatic efficiency and pressure coefficient fall off rapidly and therefore could no longer be considered constant. Using the pressure coefficient of the compressor and the fact that it remained constant at lower tip speeds, was an invaluable aid in achieving a solution to the compressor turbine unit.

As indicated in sketch (one) the compressor and turbine are connected together mechanically. The power output, from the turbine, was therefore equal to the work of compression. In order to obtain a solution to the compressor turbine unit, it was necessary to have a definite relationship between tip speed and overall pressure. This relationship presented itself in the form of the pressure coefficient of the compressor. The tip speed of the turbine and the compressor have a fixed relationship depending on the design of the unit. This coefficient remained constant within the limits of the compressor operating range and its substitution into the centripetal turbine equation proved very satisfactory for this particular power requirement. See appendix (D) for complete derivation of compressor turbine equation.

The following equation resulted:

$$\frac{W(\omega r)_t}{A_t\ P_4} = kg\sqrt{\frac{2}{k-1}}\left[\frac{C_{pc}\ kg\ R\ K_1K_2}{C_{pt}\ e_c\ e_t} - \frac{k-1}{2}\right]^{\frac{1}{2}} \frac{\omega^2 r^2}{kg\ RT_4} \times$$

$$\left[1 - \frac{\omega^2 r^2}{kg\ RT_4}\left(\frac{C_{pc}\ kg\ R\ K_1K_2}{C_{pt}\ e_c\ e_t} - \frac{k-1}{2}\right)\right]^{\frac{1}{k-1}}$$

...... Equation (2.3)

As can be seen, one of the variables, pressure ratio, has been eliminated from the turbine equation. This results in one curve of $\left(\frac{W\omega r}{A_t P_4}\right)$ versus $\left(\frac{\omega^2 r^2}{kg\ RT_4}\right)$ when the proper value of the constant is determined. The constant K_1 relates turbine tip speed and compressor tip speed. K_2 is a function of the pressure coefficient and is a constant. In this analysis, average values of k and C_p were used, depending on the temperature range. The adiabatic efficiency of the compressor is constant within its range of operation. The turbine adiabatic efficiency was assumed constant. (See Section III for discussion of turbine efficiency.)

In deriving equation (2.3) another very useful equation resulted, namely:

$$\frac{P_5}{P_4} = \left[1 - \frac{C_{pc}\ kg\ R\ K_1K_2}{C_{pt}\ e_c\ e_t} \quad \frac{\omega^2 r^2}{kg\ RT_4}\right]^{\frac{k}{k-1}}$$

... Equation (2.4)

Equation (2.4) relates the pressure drop $\left(\frac{P_4}{P_5}\right)$ and $\left(\frac{\omega^2 r^2}{kgRT_4}\right)$ required to do the compression work. Using equations (2.3) and (2.4) a simple graphical solution is possible for the compressor turbine unit. For the working curves of equations (2.3) and (2.4) see figure (5). For graphical method of solution see appendix (F).

Knowing the pressure drop in turbine (one), the remaining pressure to turbine (two) is related as follows:

$$\frac{P_6}{P_7} = \frac{P_2}{P_1} \times \frac{P_5}{P_4} \qquad \text{...... Equation (2.5)}$$

Equation (2.5) assumes no pressure loss within the cycle. In this analysis, the effect of pressure loss was neglected.

The output of the power turbine was determined as a function of the speed cubed. This type of speed - power requirement is demanded by marine propulsion plants. Having the weight flow and pressure ratio set by the compressor turbine unit, the two remaining variables of speed and temperature can be found from the turbine equation. Designing the turbines at the maximum load point and working in ratios of speed and power, the correct operating conditions at any part load may be determined by a trial and error method.

It was previously decided, to maintain the temperature constant at the power turbine and to control the cycle output by varying the burner temperature to turbine number (one).

The following, is a short brief on the method used in solving the cycle part load characteristics. For complete solution see appendix (F).

Maximum weight flow and overall pressure ratio was determined from the compressor characteristics. Maximum values of impeller tip speed and temperature were fixed, enabling one to solve for the nozzle throat area at maximum conditions. Minimum conditions of turbine number (one)were set by selecting a tip speed of 800 ft. per second. This is not the absolute minimum value, but will give a small overall pressure ratio in the compressor, which will result in a very small net output. This minimum value of tip speed fixes the range of weight flow possible from the compressor. The minimum value of weight flow was chosen so that the temperature at this reduced load would be approximately 1100°F in burner number (one). Intermediate operating points were selected in the same manner. Having a temperature schedule for burner number (one), weight flow and available pressure to turbine number (two), the problem resolved to fit these flow characteristics to the power output schedule. This resulted in a trial and error solution. With temperature to turbine number (two) fixed, it was necessary to vary the given weight flow to fit the output characteristics. Having determined the new weight flow schedule, another trial and error solution resulted, in order to find the required temperature speed schedule for turbine number (one). The final solution, however, did not effect the compressor weight flow speed characteristic curve originally chosen, therefore, the unit remained well within its stability limits. The cycle as arranged in sketch (one) and with burner number (two) operating at a fixed temperature has one and only one temperature schedule for burner number (one). Any change in burner number (one) temperature will result in a definite change of power output in turbine number (two), therefore, the cycle is temperature sensitive and our temperature control method should work very well.

III

RESULTS AND DISCUSSION

A. The Centripetal Turbine:

Evaluating the operating characteristics of the centripetal turbine from equation (2.1) is indeed a complex proceedure. Since four variables; weight flow, pressure ratio, temperature, and tip speed are present, it is possible to plot the equation in numerous combinations by fixing any two variables.

The authors were primarily interested in the effect of temperature on weight flow for a given pressure ratio and tip speeds. For fixed values of $\left(\frac{P_3}{P_1}\right)$ and (ωr), equation (2.1) will yield curves of $\left(\frac{W}{A_2 P_1}\right)$ versus (T_1).

For overall pressure ratios of 1.5 and 2.0 and tip speeds of 1000 ft/sec, the flow characteristics of the turbine are as shown in figure (6). As illustrated in the curve, the weight flow remains almost constant within the range of usual operating temperatures; therefore, the weight flow can be considered as independent of the temperature, when tip speed and pressure are fixed. This characteristic is due principally to the fact that a rise in temperature reduces the gas density in the impeller and consequently reduces the centrifugal pressure. This leaves a greater pressure drop for the nozzles which tends to increase the flow and compensate the effect of the original temperature increase.

This characteristic is of prime importance to the part load characteristics of a gas turbine cycle when the output is controlled by temperature. In effect, we have a variable area nozzle, the effective area of which is proportional to the temperature. This characteristic tends favorably toward greater stability in the compressor. In a conventional axial flow turbine with fixed nozzles, an increase in temperature reduces the weight flow which causes the compressor to momentarily operate in an unfavorable condition. Since the stability limits of compressors is a very small region, this characteristic of the centripetal turbine eliminates the stability problem involved when the power output is varied. This one characteristic alone, is of enough importance to justify its use in a gas turbine cycle, because it enables the use of temperature control, without the usual compressor stability problem.

This variable area nozzle characteristic will also cause the turbine to have very short acceleration periods. Since the weight flow remains constant with increase in temperature, the BTU per sec per pound of air entering the turbine, will increase directly with temperature. This additional energy will immediately accelerate the turbine until load conditions are again satisfied. This means that rapid changes in power output can be achieved, simply by controlling the temperature of the entering gases.

In order to interpret equation (2.1) of the turbine correctly, it is necessary to determine the limits imposed on the equation as derived. The limiting value of Mach number at the nozzle throat will be equal to (one). If the turbine is designed to operate at a condition of pressure and temperature which will result in a Mach number of (one) at the nozzle exit, the characteristic of constant weight flow with increasing temperature will no longer be true. This is evident from the fact that as nozzle discharge pressure decreases, the throat pressure approaches the critical value and then remains constant. This limiting condition has been called "The Acoustic Choke Effect". Referring to appendix (B), equation (15) relates the variables effecting acoustic choke.

$$M_2{}^2 = \frac{2}{k-1} \left[\frac{1}{\frac{k-1}{2} \; \frac{\omega^2 r^2}{kgRT_1} \quad \left(\frac{P_3}{P_1}\right)^{\frac{k-1}{k}}} - 1 \right] \quad \text{...... Equation (3.1)}$$

From the above equation one can plot values of $\left(\frac{P_3}{P_1}\right)$ versus $\left(\frac{\omega^2 r^2}{kgRT_1}\right)$ for M_2 equal to one. This curve is shown in figure (7) and indicates the maximum overall pressure drop per stage possible for a given value of $\left(\frac{\omega^2 r^2}{kgRT_1}\right)$. It is evident from figure (7) that this type of turbine is limited to low pressure ratios, and is consequently classed as a low pressure turbine, however, this limiting value of pressure drop is per stage and if it is desired to use higher pressure drops, the designer must resort to multistaging. When designing a turbine, the pressure drop per stage must not exceed the value given in figure (7) for a given temperature and tip speed. If this value is exceeded, the turbine equation will yield erroneous results.

A very interesting characteristic of this type of turbine, is the high value of adiabatic efficiency. The value of efficiency obtained on the turbines which have been built was approximately ninety percent. This value of efficiency may appear to be astonishingly high, but when the two elements of the turbine are considered, namely the nozzle and impeller, we have combined two highly efficient working elements. The test data available to the authors was conducted on normal aircraft superchargers operating as centripetal turbines. The authors did not make a theoretical investigation of turbine efficiencies, but investigations which have been made, predict an adiabatic efficiency as high as ninety five percent.[1] The efficiency of a radial turbine is a function of nozzle exit velocity and tip speed the same as in a conventional turbine, however, the overall efficiency with respect to load conditions is very flat over a considerable operating range. When temperature control is used, the characteristic U/c value for the turbine remains approximately constant. This is also true of axial flow turbines. The authors have no data, test or theoretical, to confirm the above statements. Verbal confirmation was given by persons directly connected with initial testing of this type of turbine. In view of the above facts, an overall efficiency of eighty-five percent was chosen for the cycle analysis and assumed to remain constant when operating at fractional loads.

In reviewing the assumptions made in deriving the turbine equation, the first approach was made as simple as possible. It was the intentions of the authors to correct the intitial assumptions, but time did not permit any further revisions. It is recommended that the turbine equation be corrected as follows:

1. Correct the solid vortex impeller theory for radial flow.

2. Correct for differences in Mach number at nozzle exit and impeller inlet.

3. With radial flow, nozzle angles other than zero must be investigated.

4. Frictional effects should be included.

B. Cycle Characteristics:

One of the objectives of this paper was to determine the part load characteristics of a cycle using radial type turbines. Since the radial turbine favors the temperature control method for varying power output, the cycle was arranged with this point in view. Refering to sketch number (one) the simple Brayton reheat cycle with regeneration was the type chosen for this analysis. The cycle control point is at burner number (one) and by regulating the fuel to this burner, the net cycle output can be controlled.

The problems which confronted the authors in making this first cycle analysis were numerous, and a great amount of the time was consumed in determining a method of solution for the given cycle. The method of solution outlined in appendix (F) is only one of the many possible ways the problem can be solved. A discussion of methods of solution for part load characteristics will not be undertaken in this paper, because of the complexity of the problem involved. The authors feel very fortunate in having found the trial and error method of solution which is discussed fully in appendix (F).

In designing the turbine, the maximum load conditions were assumed to be 100% . Maximum conditions of temperature and tip speed were limited to present metallurgical standards and maximum pressure ratios were determined by the acoustic choke effect. The double flow two stage compressor was selected to produce an overall pressure ratio of (five) so that the cycle would be operating at its maximum efficiency when temperature equaled 1500°F and a regenerator of 75% effectiveness was used. In this cycle analysis no corrections were made for variation of regenerator effectiveness with load or of pressure losses within the cycle.

The cycle characteristic of greatest interest are represented in figure (8). Overall cycle efficiency expressed in percent of full load efficiency is plotted against percent maximum load. Figure (8) also shows the same curve for a conventional turbine cycle using the temperature control method. As can be seen, the centripetal turbine has higher fractional load efficiencies than could be obtained in a conventional turbine cycle. Although the cycle efficiency for the centripetal turbine is higher than an axial flow turbine at part load conditions, the characteristic of

greatest importance is the cycle stability when undergoing load changes. The weight flow thru the centripetal turbine will not be effected by temperature changes, therefore, the compressor will operate along its designed operating curve. The possibility of forcing cycle instability when the load is changed, is negligable, because there is no choking off of weight flow, when the temperature is increased. In conventional turbines with fixed nozzles, this characteristic is not present and all load changes must be made slowly so that the compressor will not be forced into an unstable operating region.

Figure (9) shows the temperature schedule for the various operating points in the cycle, when operating at any part load condition. As indicated in figure (9), there is a part load condition when the temperature from the regenerator exceeds the temperature required for turbine number (one). For part loads below this value, several methods can be used. One apparent method would be to dump some of the gases before they reach the regenerator, thereby lowering the gas temperature leaving the regenerator. As indicated in figure (9), the temperature schedule to turbine number (one), the control temperature increases uniformly with increase in load. This indicates that the cycle is temperature sensitive and will respond rapidly to any variation in the above temperature. This is an essential characteristic for any marine propulsion cycle.

Figure (10) shows the weight flow and speed characteristics of the cycle of fractional load conditions. The variation is uniform with increasing load thus giving a good indication of stable performance at any load condition.

The cycle characteristics at maximum load condition were as follows:

1. Cycle efficiency 36%
2. Power output . . . . 1380 H.P.
3. Turbine tip speed . . . . 1200 ft/sec.
4. Turbine nozzle temperature . . . . 1500°F
5. Weight flow . . . . 13.0 lbs/sec
6. Overall pressure ratio 4.9
7. Compressor tip speed . . . . 1020 ft/sec
8. Compressor adiabatic efficiency . . . 80%
9. Turbine adiabatic efficiency 85%

If the above turbine cycle was installed in a typical destroyer escort hull, with speed power characteristics as shown in figure (11), the speed range available by the control method used would be from 12 to 20 knots. This is assuming that a power turbine was installed on each shaft. For speeds lower than 12 knots the temperature to turbine number (two) could be reduced and a schedule of dumping exhaust gases set up to keep the required temperature schedule on turbine number (one).

Other apparent advantages of the centripetal turbine are as follows:

1. Radial turbine will be insensitive to dirt and small damages because none of the construction parts have a special aerodynamic shape. Production costs for single stage radial turbines will undoubtedly be lower than that for axial turbines of the same output.

2. The 90° impeller blade can be a light and relatively inexpensive construction part. Low impeller weight results in a thin shaft and thus in a low total runner weight and inertia moment. Accordingly, low acceleration times are obtained when in need of quick changes of R.P.M.

3. Pivoted nozzles may readily be adapted thus offering a decided advantage when changes in operating conditions are necessary.

IV

CONCLUSIONS AND RECOMMENDATIONS

1. The centripetal gas turbine has an inherent characteristic of variable nozzle area, when using the temperature control method.

2. The above characteristic gives greater cycle stability when load conditions are changed.

3. Short acceleration periods are obtained when in need of quick changes of R.P.M.

4. The adiabatic efficiency of the turbine is high and it possesses a flat efficiency curve with varying loads.

5. The radial turbine is inherently a low pressure turbine when used as a single stage unit.

6. The turbine as employed in a marine gas turbine cycle proved flexible enough to allow a speed range of 12 to 20 knots, in a 1914 destroyer of the U.S.S. Wadsworth DD60 type. This speed range was attained by temperature control alone and lower speeds can easily be attained by decreasing the temperature to turbine number (two).

7. The gas turbine cycle had a maximum efficiency of 36% at full load and 18% efficiency at 22% of full load.

8. The radial turbine will be insensitive to dirt and light damage, because it possesses no special aerodynamic shape.

9. Production costs for a single stage radial turbine will undoubtedly be lower than that for axial turbines of the same output.

Recommendations:

1. The centrifugal turbine theory be corrected for radial flow, variation in Mach number and nozzle angle effect.

2. Regeneration and pressure loss effects be included in cycle computations.

3. Effect of varying temperature of both turbines with changing load conditions.

4. A complete study, using two dimensional flow theory be made to determine limits of efficiency and blade design characteristics, such as we now have for centrifugal compressors.

V

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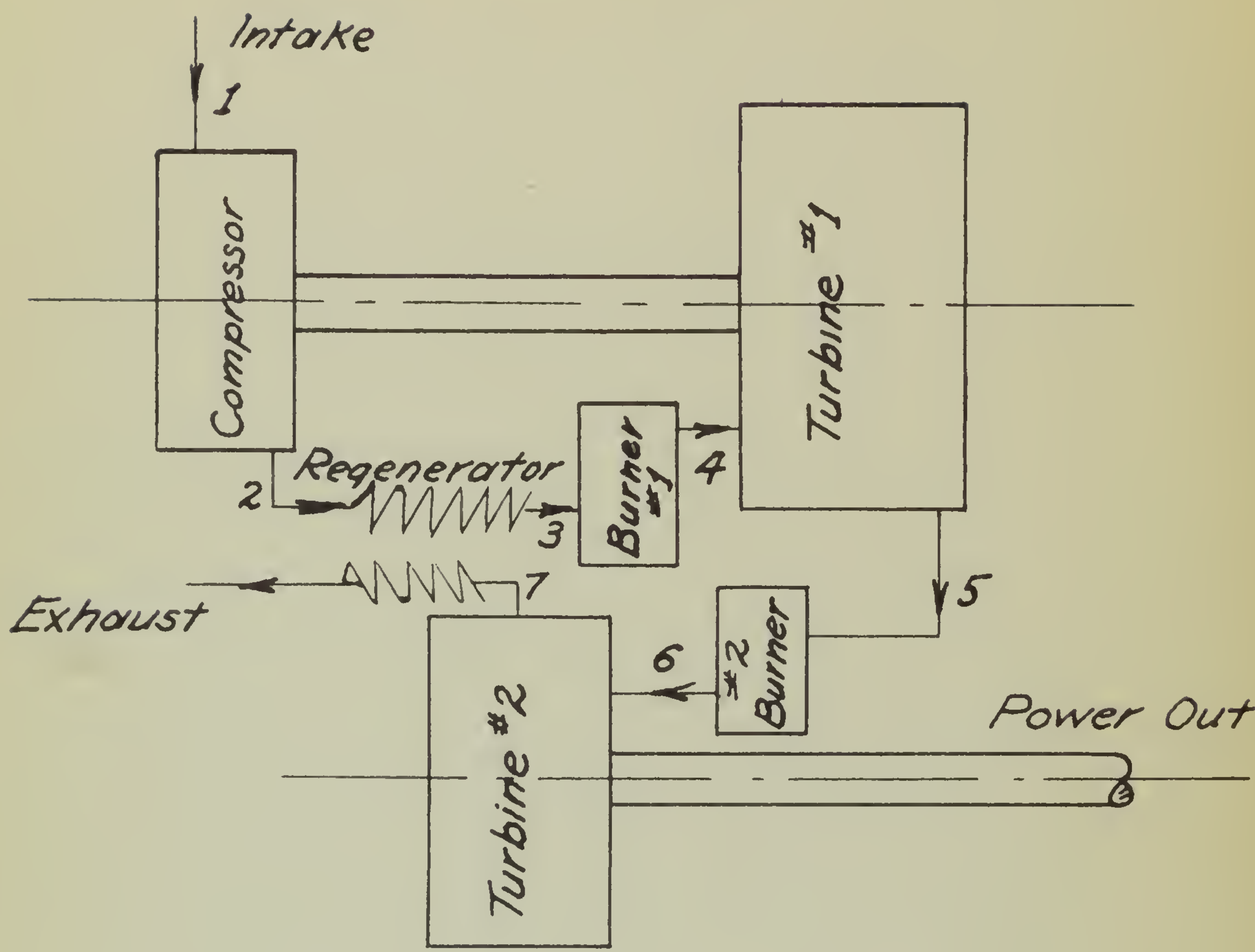
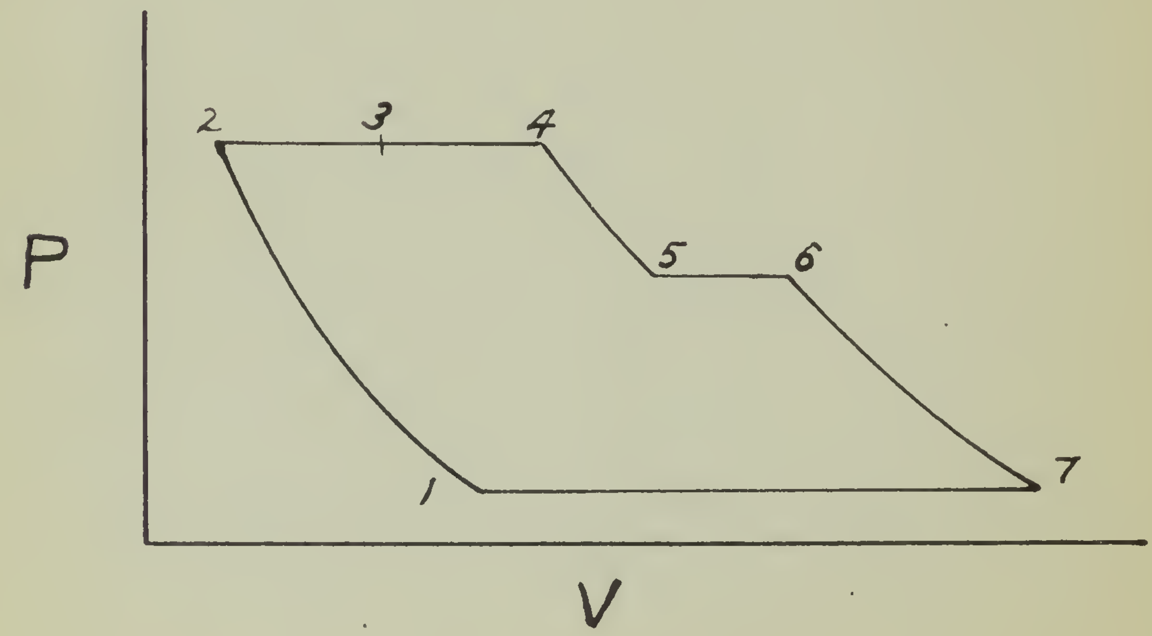
SKETCH I
CYCLE





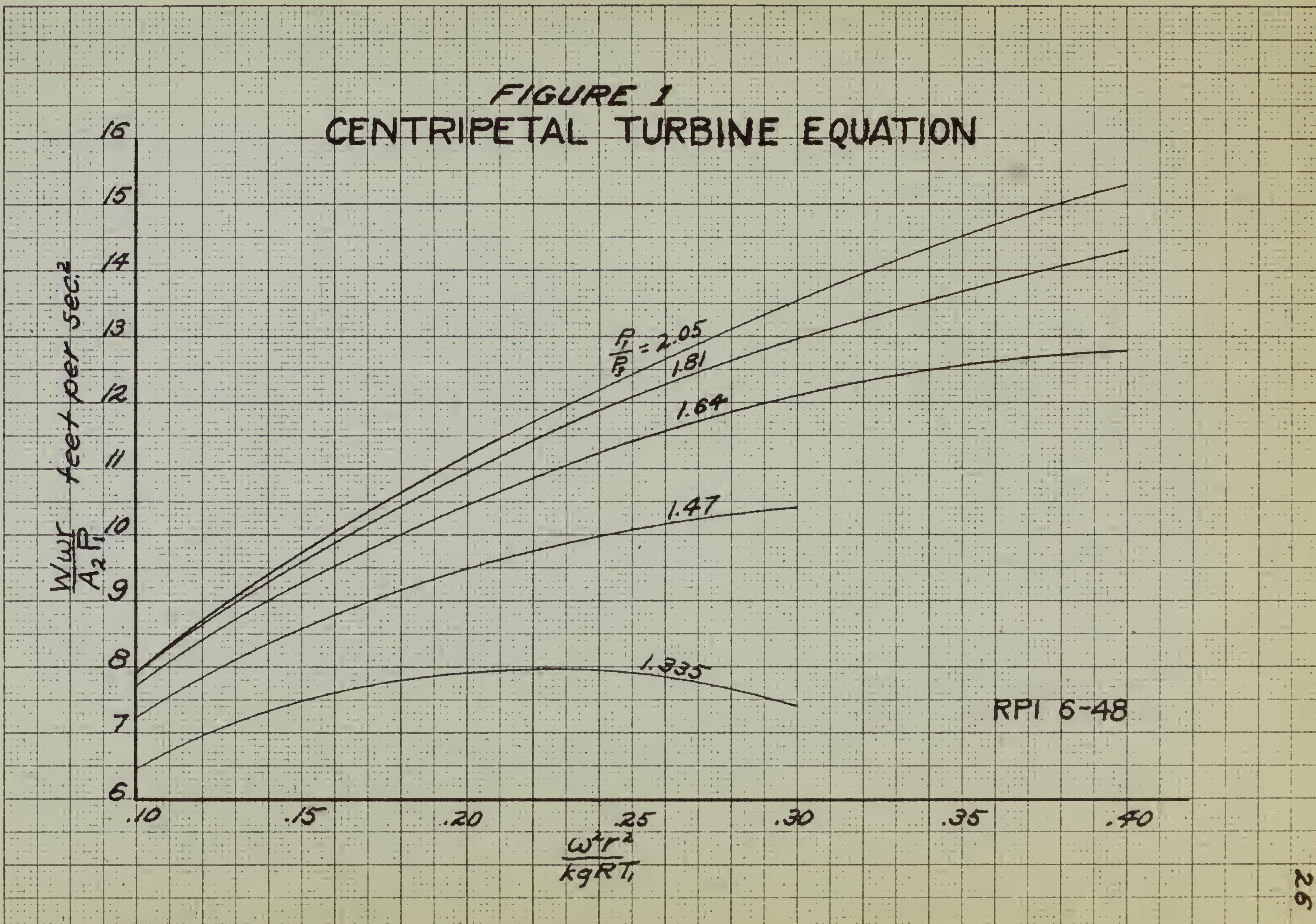


FIGURE 1
CENTRIPETAL TURBINE EQUATION
$\frac{W\omega r}{A_2 P_1}$ feet per sec.²
16
15
14
13
12
11
10
9
8
7
6
.10
.15
.20
.25
.30
.35
.40
$\frac{\omega^2 r^2}{kgRT_1}$
$\frac{P_1}{P_3}$ = 2.05
1.81
1.64
1.47
1.335
RPI 6-48

FIGURE 2
CENTRIFUGAL COMPRESSOR
Pressure Ratio Vs. Tip Velocity

RPI 6-48

FIGURE 3
CENTRIFUGAL COMPRESSOR
Stability Limits

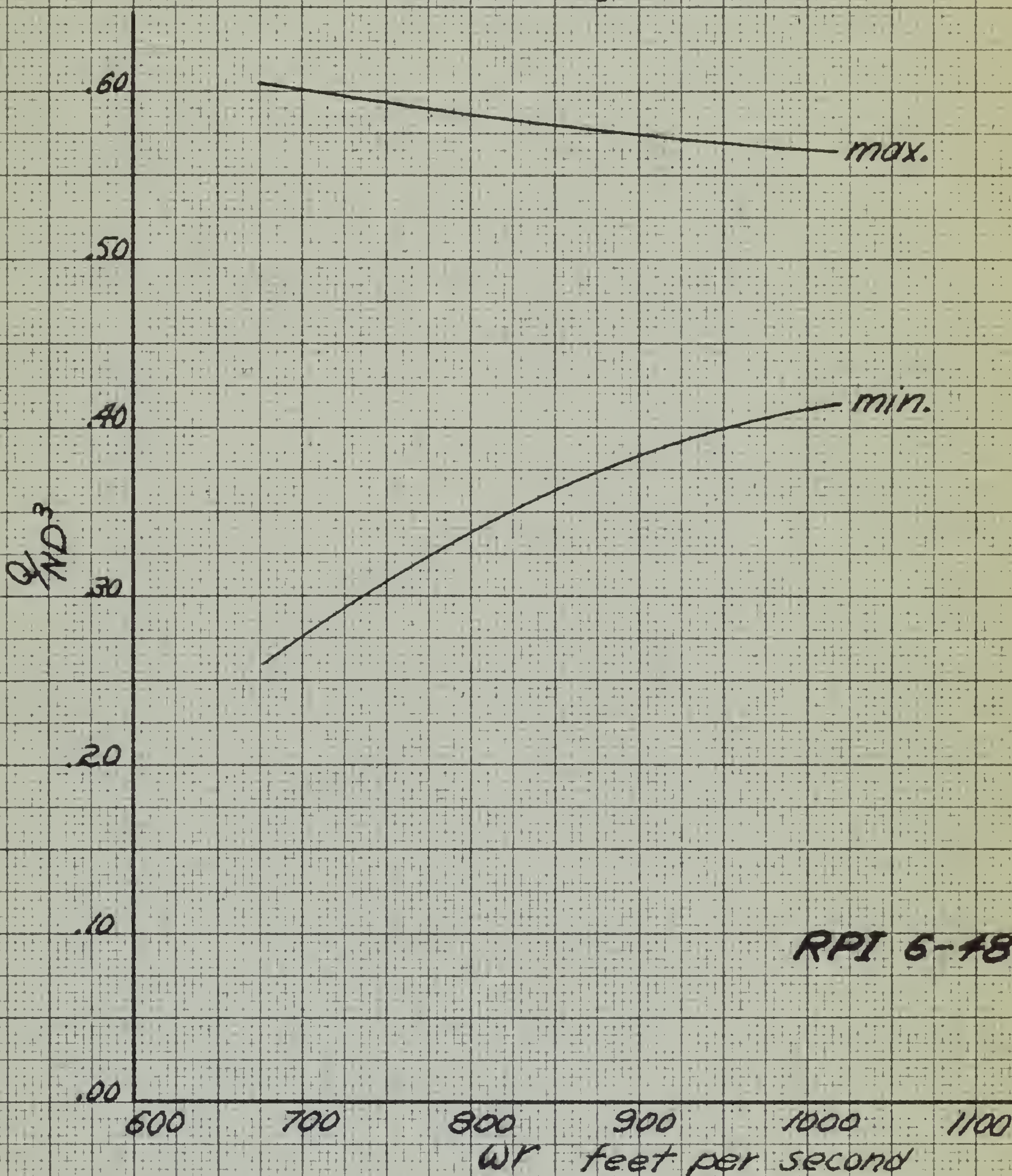
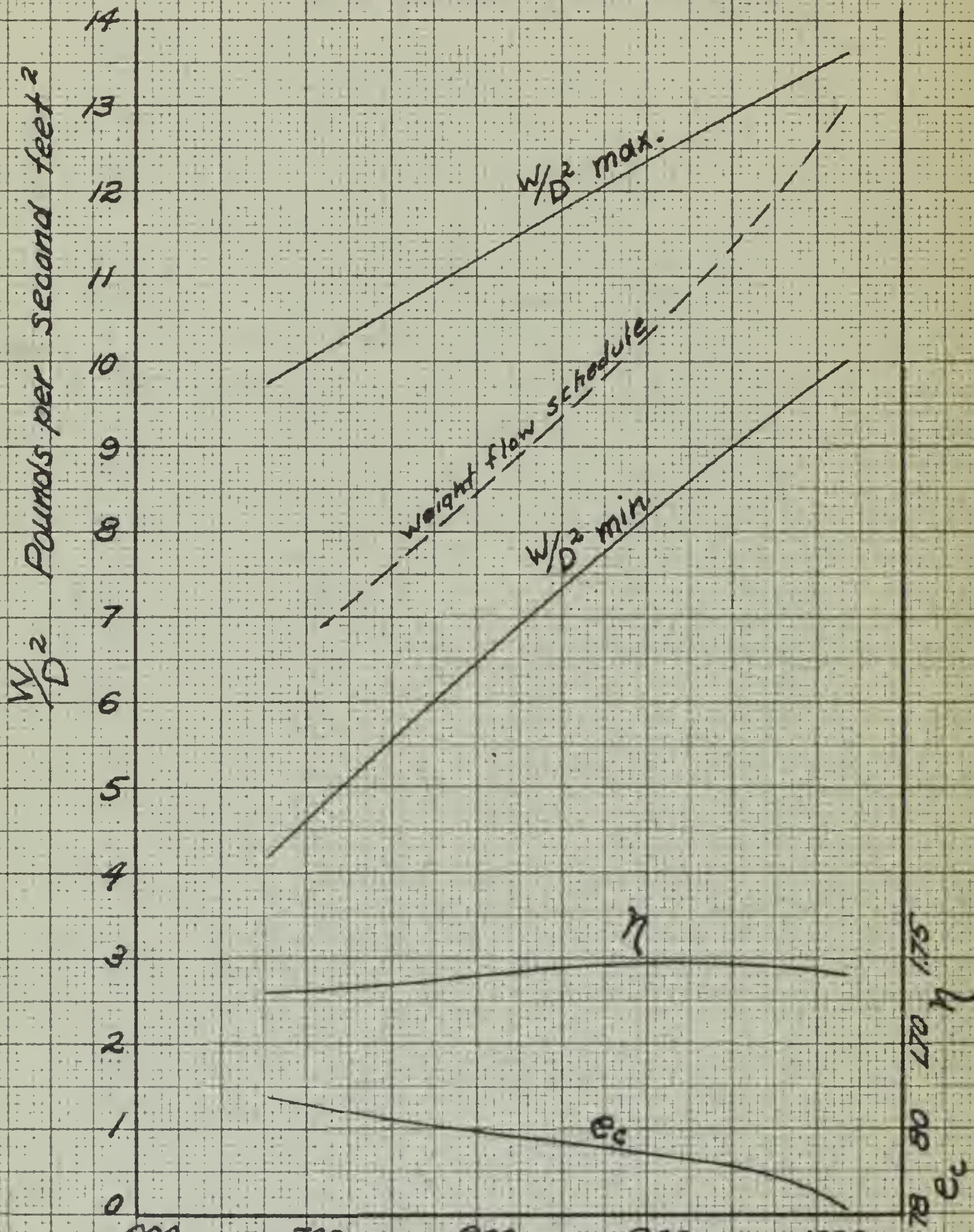


FIGURE 4
CENTRIFUGAL COMPRESSOR CHARACTERISTICS



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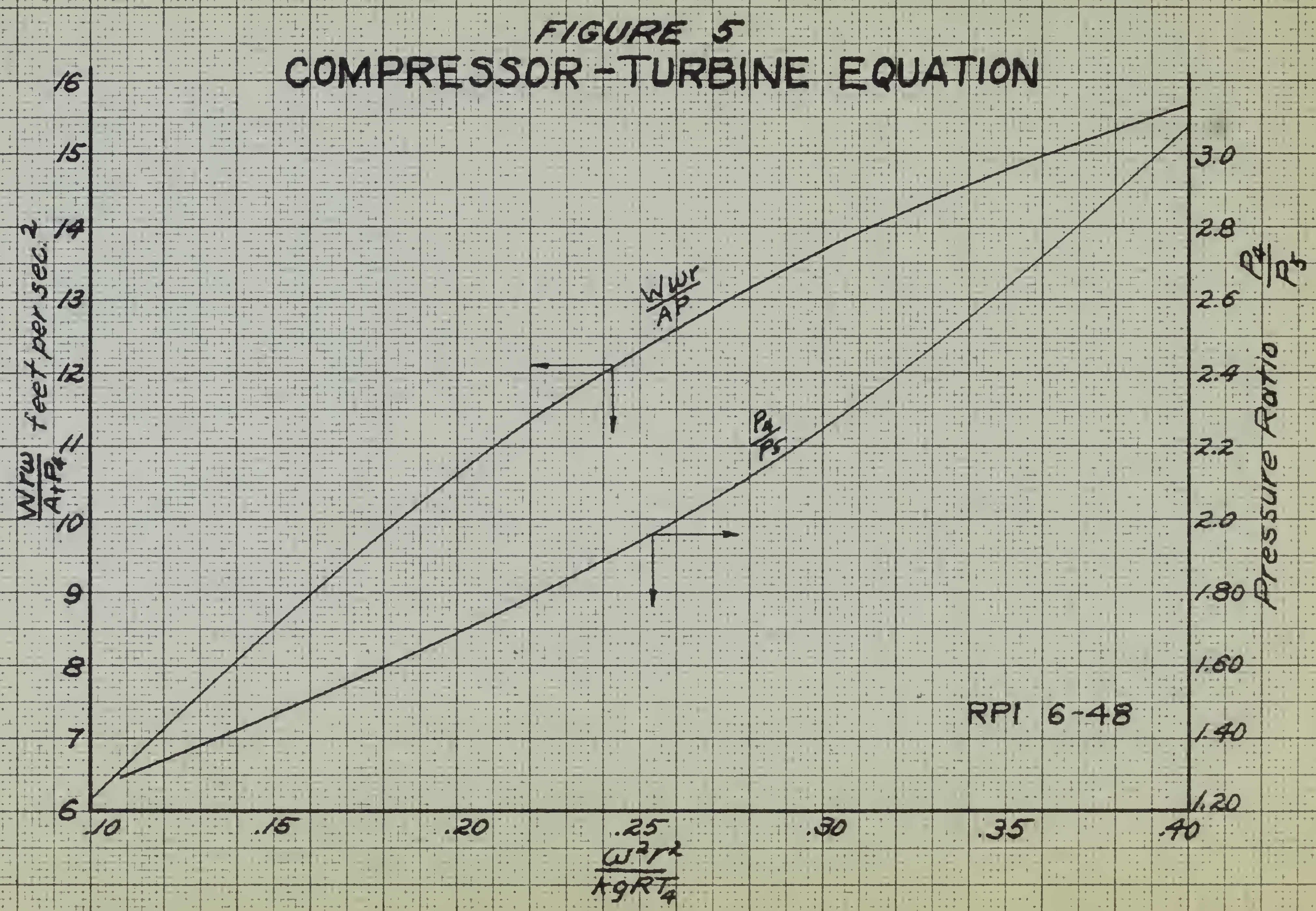


FIGURE 5
COMPRESSOR - TURBINE EQUATION
16
15
14
13
12
11
10
9
8
7
6
$\frac{W r \omega}{A_t P_4}$ feet per sec.^{2}
.10
.15
.20
.25
.30
.35
.40
$\frac{\omega^2 r^2}{k g R T_4}$
3.0
2.8
2.6
2.4
2.2
2.0
1.80
1.60
1.40
1.20
Pressure Ratio $\frac{P_4}{P_5}$
$\frac{W \omega r}{A P}$
$\frac{P_4}{P_5}$
RPI 6-48

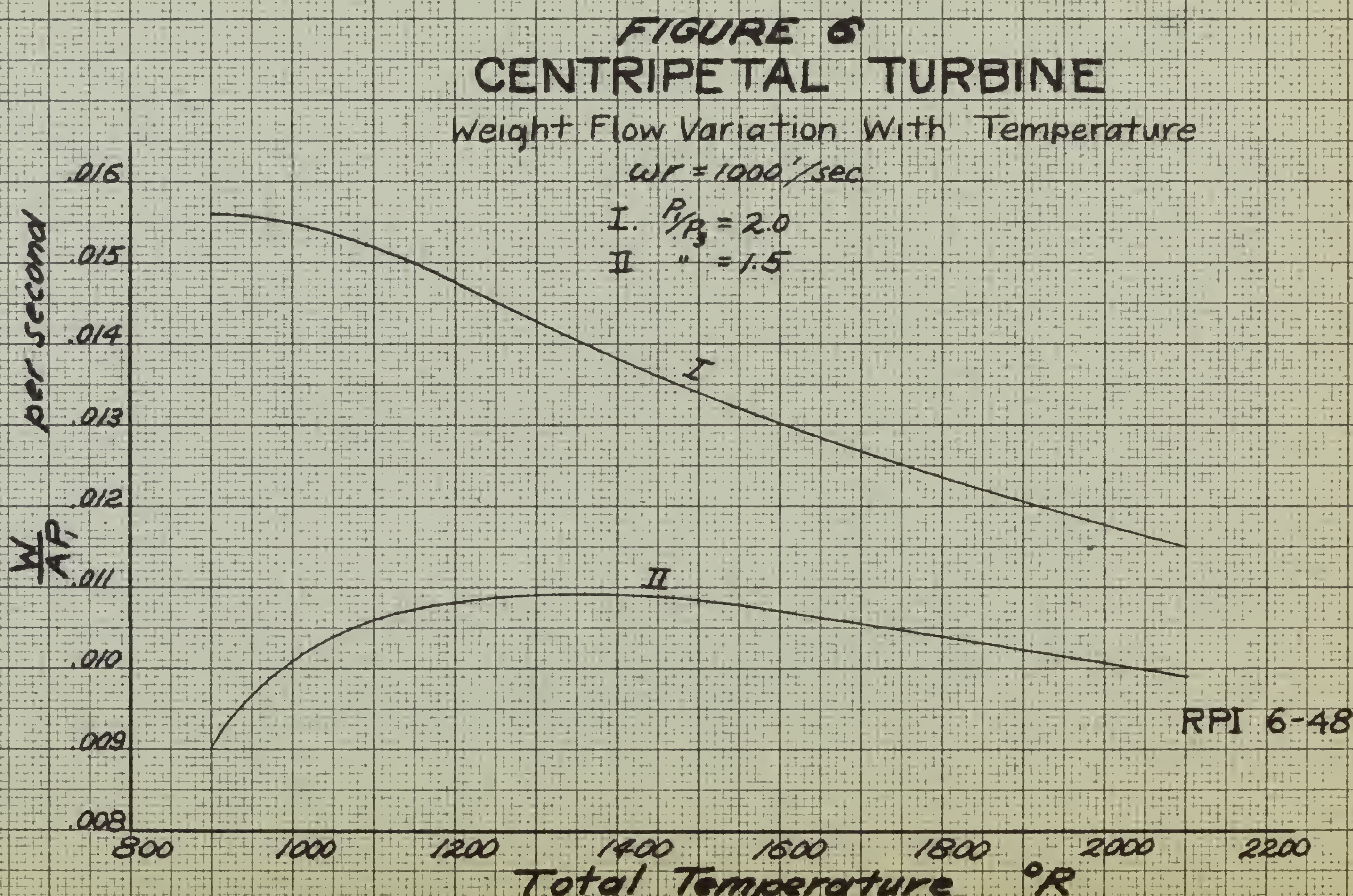


FIGURE 6
CENTRIPETAL TURBINE
Weight Flow Variation With Temperature
ωr = 1000'/sec
I. P_1/P_3 = 2.0
II " = 1.5
I
II
$\frac{W}{AP_1}$ per second
.016
.015
.014
.013
.012
.011
.010
.009
.008
800
1000
1200
1400
1600
1800
2000
2200
Total Temperature °R
RPI 6-48

FIGURE 7
ACOUSTIC CHOKE LIMIT
Equation 3.1
$K=1.340 \quad M_2=1.0$

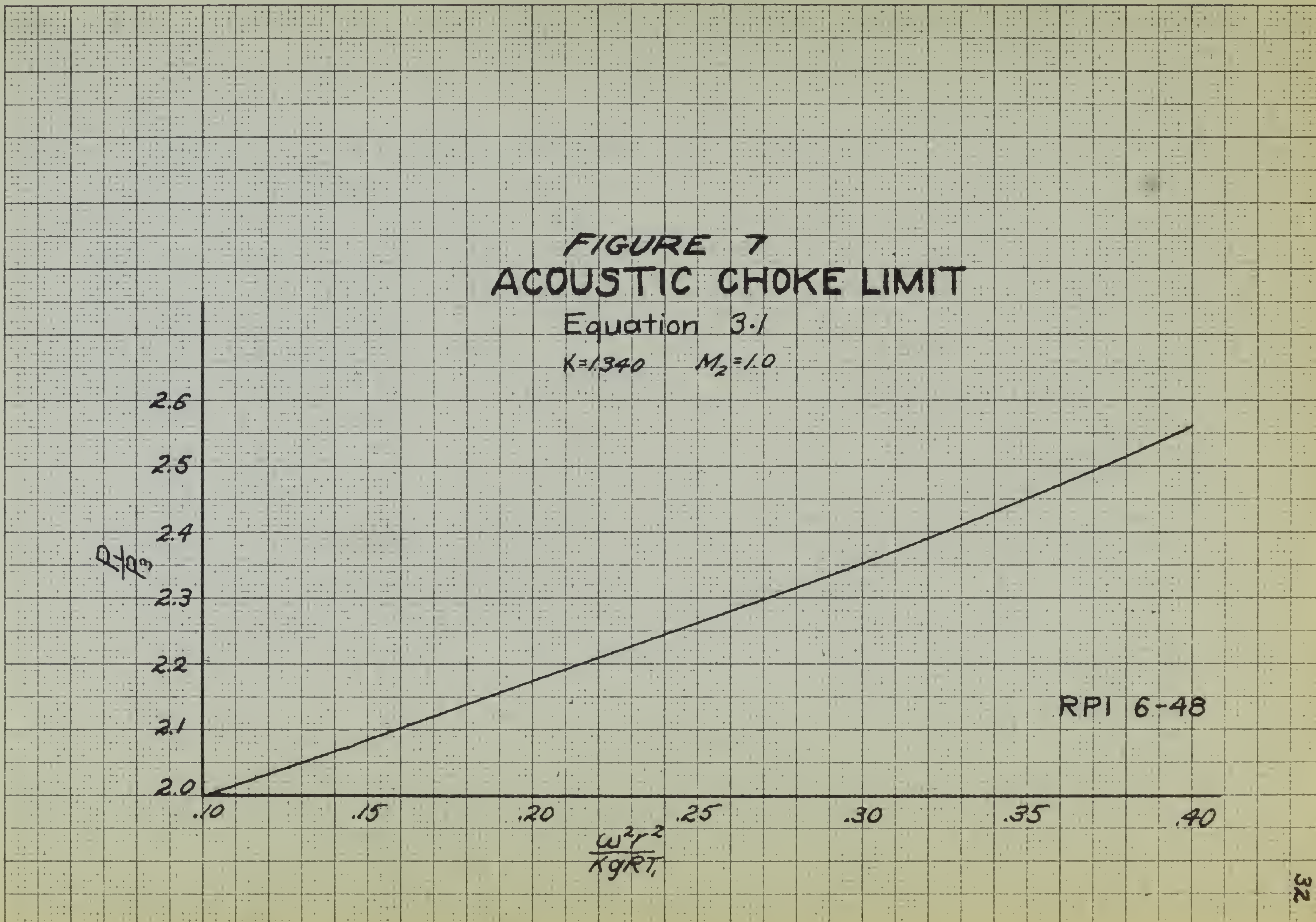


FIGURE 8
CYCLE EFFICIENCY

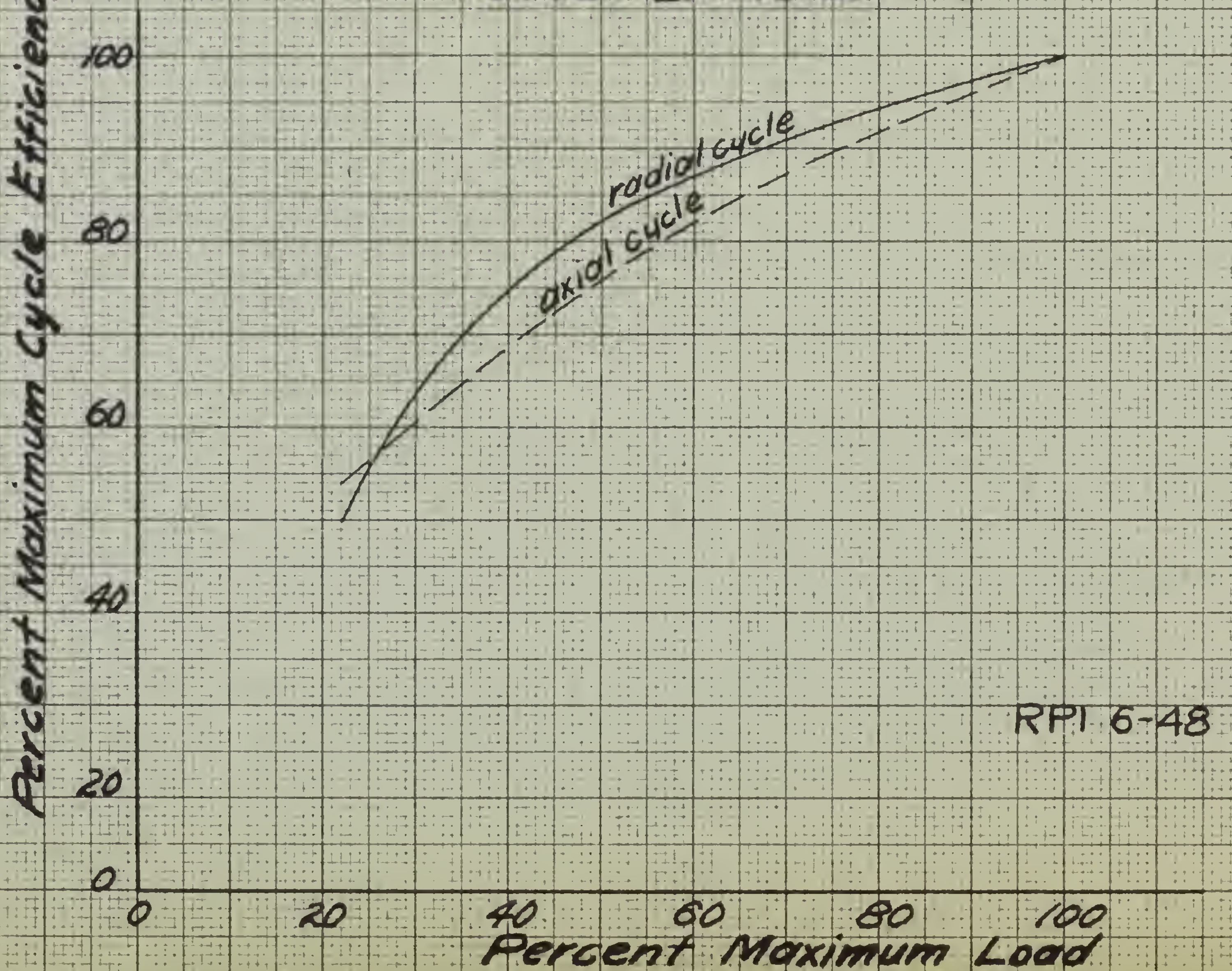
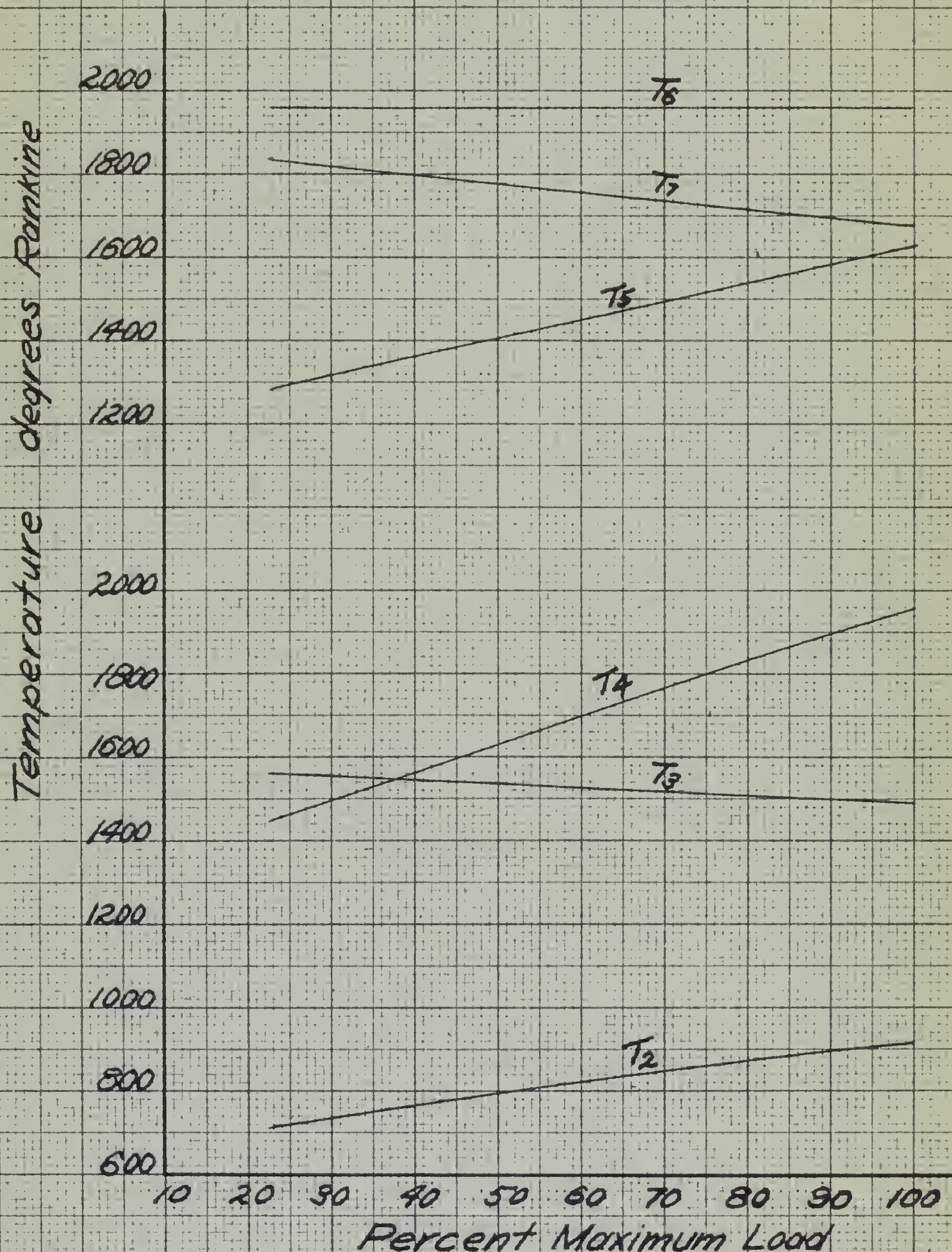


FIGURE 9
CYCLE TEMPERATURES
AT VARIOUS LOADS



RPI 6-48

FIGURE 10
CYCLE SPEED AND WEIGHT FLOW

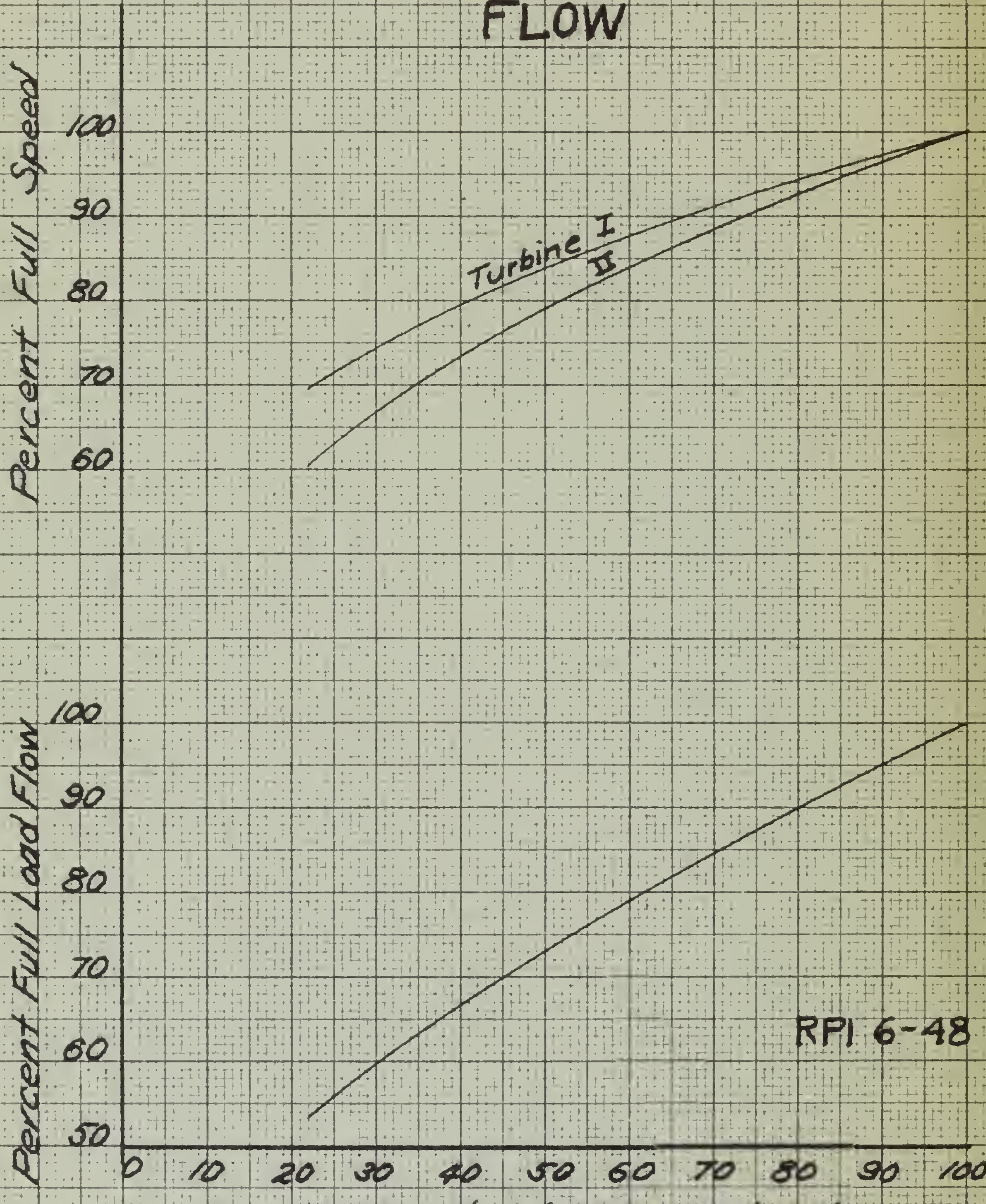
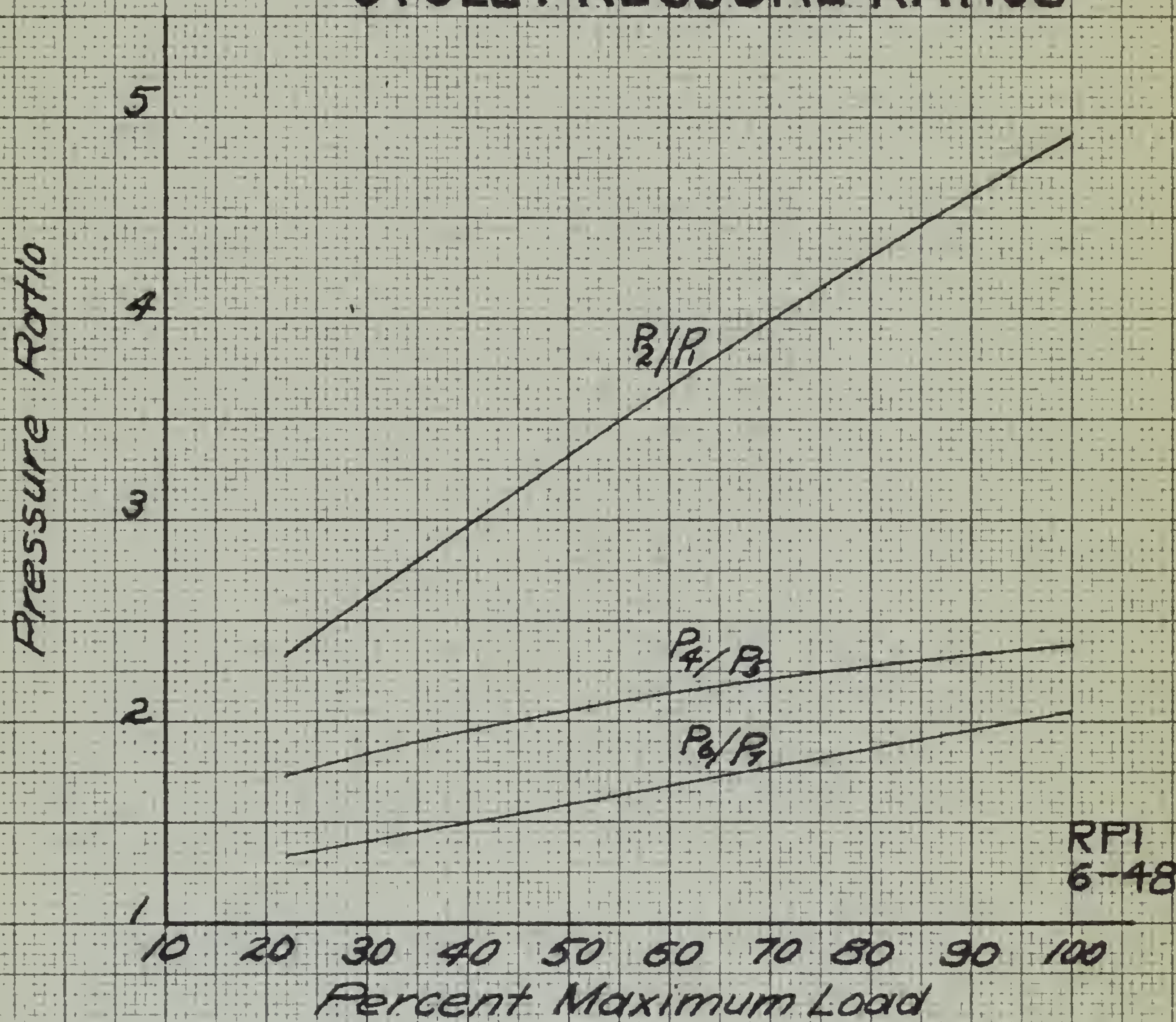


FIGURE 11

CYCLE PRESSURE RATIOS



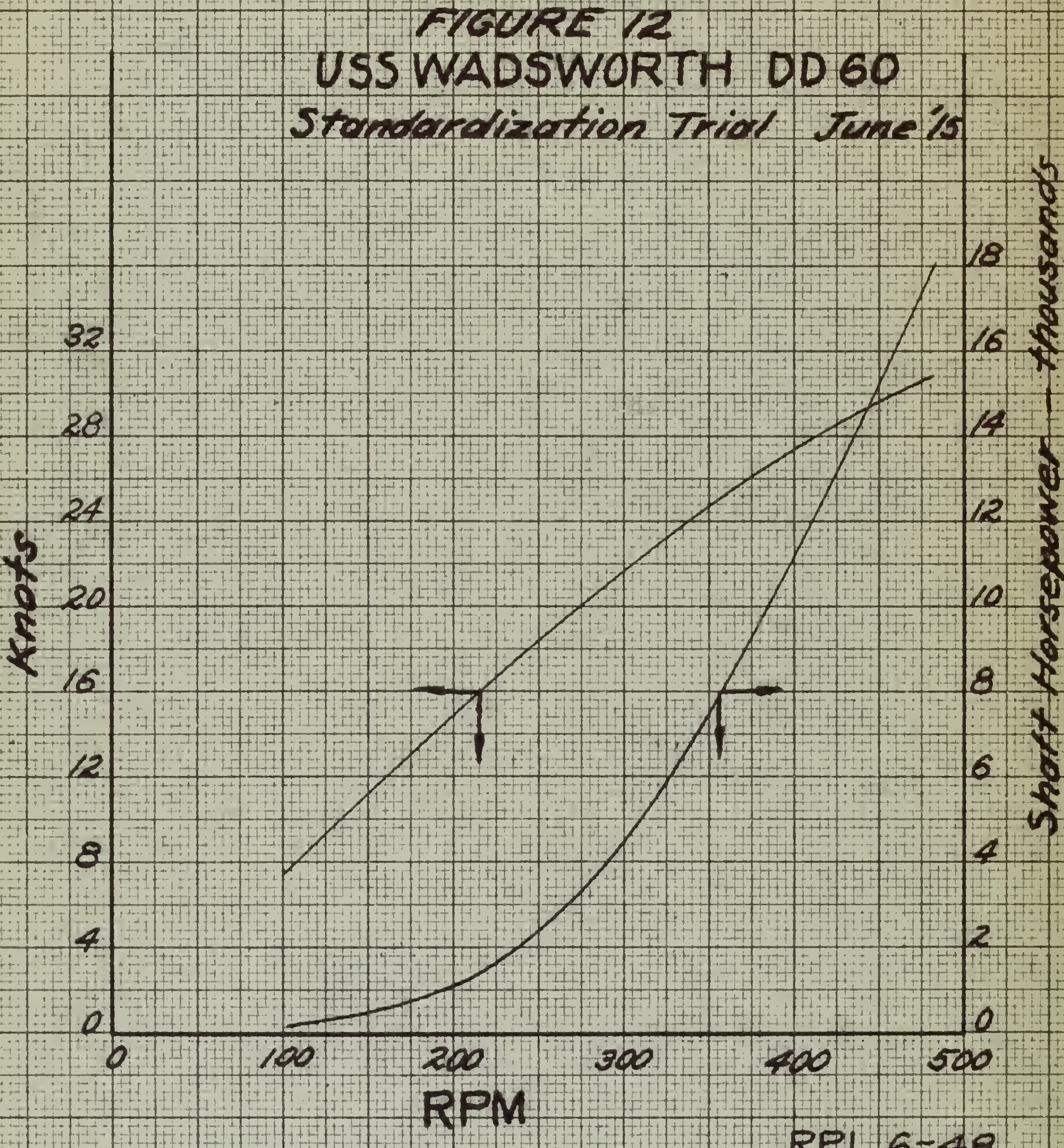


FIGURE 12
USS WADSWORTH DD 60
Standardization Trial June '15
Knots
0
4
8
12
16
20
24
28
32
Shaft Horsepower — thousands
0
2
4
6
8
10
12
14
16
18
0
100
200
300
400
500
RPM
RPI 6-48

TABLE I

Centripetal Turbine Equation

No.	$\frac{\omega r^2}{kgRT_6}$	$\frac{W \omega r}{A_t P_6}$ $\frac{P_6}{P_7} = 2.05$	1.81	1.64	1.47	1.335
1	.10	7.95	7.88	7.75	7.24	6.47
2	.15	9.74	9.61	9.30	8.61	7.50
3	.20	11.20	10.96	10.49	9.53	7.96
4	.25	12.45	12.10	11.42	10.10	7.97
5	.30	13.55	13.00	12.11	10.40	7.45
6	.35	14.52	13.75	12.59		
7	.40	15.30	14.29	12.80		

(See Figure 1)

TABLE II

Centrifugal Compressor Characteristics.

No.	1	2	3	4	5	6	7
$r^2\omega^2/gRT$	.90	.85	.80	.75	.70	.65	.60
$r\omega$ ft/sec	1017	960	904	847	791	734	677
P_2/P_1	4.88	4.24	3.68	3.24	2.82	2.45	2.16
$\frac{Q}{ND^3}$ Max.	.564		.575		.588		.604
$\frac{Q}{ND^3}$ Min.	.414		.386		.332		.260
$\frac{W}{D^2}$ Max.	13.62		12.40		11.10		9.74
$\frac{W}{D^2}$ Min.	10.00		8.31		6.25		4.20
e_c %	78.1		79.4		80.0		80.8
η	1.741		1.748		1.738		1.730

(See Figures 2, 3, and 4)

TABLE III

Compressor Turbine Equation

No.	$\frac{\omega^2 r^2}{kgRT_4}$	$\frac{W \omega r}{A_t P_4}$	$\frac{P_4}{P_5}$
1	.10	6.12	1.289
2	.15	8.55	1.470
3	.20	10.62	1.691
4	.25	12.32	1.941
5	.30	13.71	2.255
6	.35	14.80	2.630
7	.40	15.65	3.075

TABLE IV

Centripetal Turbine

No.	T °R	W/A_2P_1	
		$\frac{P_3}{P_1} = 1.5$	$\frac{P_3}{P_1} = 2.0$
1	900	.0090	.0156
2	1000	.0101	.0155
3	1100	.0106	.0152
4	1200	.0108	.0147
5	1300	.0109	.0142
6	1400	.0109	.0138
7	1500	.0108	.0134
8	1600	.0107	.0131
9	1700	.0105	.0127
10	1800	.0104	.0124
11	2000	.0101	.0118

TABLE V

Acoustic Choke Limit

No.	$\frac{\omega^2 r^2}{kgRT_1}$	$\frac{P_3}{P_1}$
1	.10	2.00
2	.15	2.09
3	.20	2.17
4	.25	2.27
5	.30	2.35
6	.32	2.39
7	.35	2.46
8	.40	2.56

TABLE VI

Cycle Characteristics

No.	% Max. Load	H.P.	Eff. %	% Max. Eff.	W lbs/sec	% Max. W	Turbine # 1		Turbine # 2		T_1 °R	T_2 °R
							ωr	% Max. ωr	ωr	% ωr		
1	100	1380	36.0	100	13.00	100	1200	100	1200	100	530	907
2	73.3	1011	33.2	92.1	11.25	86.5	1112	92.6	1082	90.1	530	855
3	54.0	745	30.5	84.7	9.83	75.6	1029	85.6	978	81.4	530	808
4	36.0	496	25.8	71.7	8.30	63.8	933	77.6	855	71.1	530	756
5	22.3	308	18.0	50.0	6.92	53.2	840	69.9	729	60.7	530	709

No.	T_3 °R	T_4 °R	T_5 °R	T_6 °R	T_7 °R	$\frac{P_2}{P_1}$	$\frac{P_4}{P_5}$	$\frac{P_6}{P_7}$	Eff. % Max. Axial Turbine			
1	1490	1960	1630	1960	1683	4.90	2.390	2.05	100			
2	1507	1795	1510	1960	1725	4.08	2.250	1.81	89			
3	1525	1660	1419	1960	1762	3.44	2.095	1.64	79			
4	1541	1532	1330	1960	1804	2.83	1.925	1.47	65			
5	1559	1448	1285	1960	1844	2.33	1.745	1.335	54			

Appendix A

NOMENCLATURE

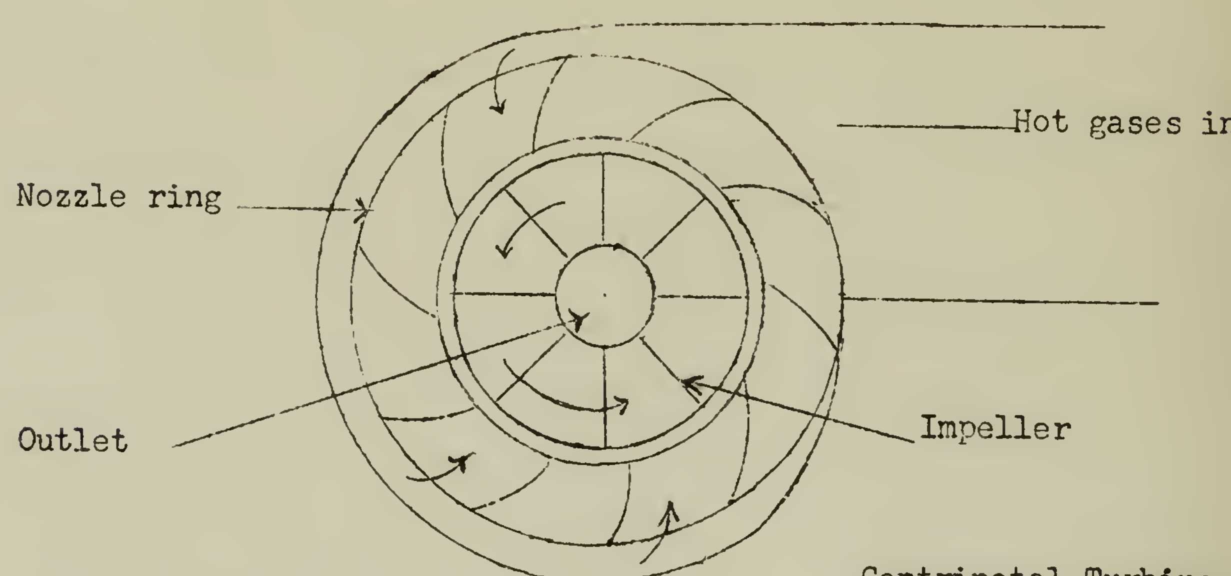
Symbol	Meaning	Units
A	Nozzle Throat Area	in^2
C_p	Specific Heat at Constant Pressure	BTU/lb/^oR
C_v	Specific Heat at Constant Volume	BTU/lb/^oR
D	Diameter Compressor Impeller	feet
e_c	Compressor Efficiency	- - -
e_t	Turbine Efficiency	- - -
f	Regenerator Effectiveness	- - -
g	Gravitational Constant	ft/sec^2
HP	Horsepower	- - -
J	Conversion Factor	ft.lbs/BTU
k	Ratio Specific Heats for Gases	- - -
K	Constant	- - -
M	Mach Number	- - -
N	Shaft RPM	revs/min
P	Static Pressure	lbs/in^2
Q	Volumetric Flow	ft^3/sec
R	Gas Constant	lb. ft/^oR
r	Impeller radius	feet
T	Total Absolute Temperature	degrees R
W	Weight Flow per second	lbs/sec
η	Pressure Coefficient	- - -
γ	Ratio specific Heats for Air	- - -
ρ	Density	slugs/ft^3
ω	Angular Velocity	radians/sec
ωr	Impeller Tip Speed	feet/sec.

Note: Subscripts denote location of the condition.
Example: $r_c \, \omega_c$ - compressor tip speed.
T_1 - temperature @ point 1..

Appendix B

CENTRIPETAL TURBINE EQUATION

To analyze the flow conditions thru a centripetal turbine it is necessary to use thermodynamic knowledge of flow thru a centrifugal compressor and a nozzle. Then attempt to derive an equation relating the following variables; overall pressure ratio, temperature entering nozzle, weight flow per second per unit nozzle exit area and the speed of the turbine.



Centripetal Turbine

The following assumptions were made:

1. Non flow, solid forced vortex impeller theory.
2. Adiabatic impeller compression.
3. Ideal nozzle (reversible)
4. Mach number at nozzle exit and impeller inlet are equal.
5. Frictional effects neglected.
6. Nozzles are set tangent to impeller circumference. (Zero nozzle angles).

The following system of subscripts were used:

1. Entrance conditions at nozzle ring.
2. Conditions at nozzle exit and impeller inlet.
3. Discharge conditions from turbine.

Impeller:

The simplest version of a centrifugal impeller is a solid forced vortex rotating at constant angular speed (ω) with the air flowing outward with a radial velocity that is negligible as compared with the tangential velocity. The pressure increase (dP) in a radial distance (dr) from summation of forces is:

$$dP = \rho\,\omega^2\, rdr = \frac{P}{gRT}\,\omega^2\, rdr \qquad (1)$$

or

$$\frac{dP}{P} = \frac{\omega^2\, rdr}{g\, RT} \qquad (2)$$

For reversible flow thru the impeller,

$$\frac{T_2}{T} = \left(\frac{P_2}{P}\right)^{\frac{\gamma-1}{\gamma}} \qquad (3)$$

From equations 2 and 3,

$$dP = \frac{P}{gRT_2}\left(\frac{P_2}{P}\right)^{\frac{\gamma-1}{\gamma}}\omega^2\, rdr \qquad (4)$$

$$P^{-\frac{1}{\gamma}}\, dP = \frac{P_2^{\frac{\gamma-1}{\gamma}}}{g\, RT_2}\,\omega^2\, rdr \qquad (5)$$

Intergrating equation 5,

$$\frac{\gamma}{\gamma-1}\left(\frac{P}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\omega^2 r^2}{2\gamma gRT_2} + C_1 \qquad (6)$$

In any actual impeller there must be a finite inlet if there is to be flow, but since an ideal inlet of an impeller is charged with replacing an equivalent forced vortex the limiting case of (6) is,

$$r = 0 \qquad P = P_3$$

therefore,

$$C_1 = \frac{\gamma}{\gamma-1} \; \frac{P_3}{P_2}^{\frac{\gamma-1}{\gamma}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

giving,

$$\frac{\gamma}{\gamma-1}\left(\frac{P}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\omega^2 r^2}{2gRT_2} + \frac{\gamma}{\gamma-1}\;\frac{P_3}{P_2}^{\frac{\gamma-1}{\gamma}} \quad \ldots\ldots\ldots \quad (8)$$

at $r = r_2$, $P = P_2$

$$\frac{\gamma}{\gamma-1} = \frac{\omega^2 r^2{}_2}{2gRT_2} + \frac{\gamma}{\gamma-1}\;\frac{P_3}{P_2}^{\frac{\gamma-1}{\gamma}} \quad \ldots\ldots\ldots\ldots \quad (9)$$

or,

$$\frac{\gamma}{\gamma-1} = \frac{\omega^2 r^2{}_2}{2gRT_2} + \frac{\gamma}{\gamma-1}\left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{P_1}{P_2}^{\frac{\gamma-1}{\gamma}} \quad \ldots\ldots \quad (10)$$

Nozzle:

From High Velocity Thermodynamics (3) for a reversible nozzle expansion,

$$\frac{P_1}{P_2} = \left[1 + \frac{\gamma-1}{2} M_2{}^2\right]^{\frac{\gamma}{\gamma-1}} \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

From High Velocity Thermodynamics the relation between total and static temperature,

$$T_2 = \frac{T_{o2}}{\left[1 + \frac{\gamma-1}{2} M_2{}^2\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

Combining equations 10, 11, and 12,

$$\frac{\gamma}{\gamma-1} = \frac{\omega^2 r^2{}_2}{2gRT_{o2}}\left[1 + \frac{\gamma-1}{2} M_2{}^2\right] + \frac{\gamma}{\gamma-1}\left[1 + \frac{\gamma-1}{2} M_2{}^2\right] \frac{P_3}{P_1}^{\frac{\gamma-1}{\gamma}} \ldots (13)$$

giving,

$$\left[1 + \frac{\gamma -1}{2} M_2^{\,2}\right] = \frac{1}{\left[\left(\frac{\gamma -1}{2}\right) \frac{\omega^2 r^2{}_2}{\gamma g R T o_2} + \frac{P_3}{P_1}^{\frac{\gamma -1}{\gamma}}\right]} \quad . . . . . (14)$$

or,

$$M_2^{\,2} = \frac{2}{\gamma -1} \left[\frac{1}{\left[\left(\frac{\gamma -1}{2}\right) \frac{\omega^2 r^2{}_2}{\gamma g R T o_2} + \frac{P_3}{P_1}^{\frac{\gamma -1}{\gamma}}\right]} - 1\right] \quad . . . (15)$$

From High Velocity Thermodynamics for a total temperature (To_2), a pressure (P_2) of the gases at the nozzle discharge area (A_2),

$$\frac{W \sqrt{To_2}}{A_2\, P_2} = M_2 \sqrt{\frac{\gamma g}{R} \left[1 + \frac{\gamma -1}{2} M_2^{\,2}\right]} \quad . . . . (16)$$

or,

$$\frac{W \sqrt{To_2}}{A_2\, P_1} = \frac{P_2}{P_1} M_2 \sqrt{\frac{\gamma g}{R} \left[1 + \frac{\gamma -1}{2} M_2^{\,2}\right]} \quad . . . . (17)$$

Using equation (11) in (17),

$$\frac{W \sqrt{To_2}}{A_2\, P_1} = M_2 \sqrt{\frac{\gamma g}{R}} \left[1 + \frac{\gamma -1}{2} M_2^{\,2}\right]^{-\frac{(\gamma + 1)}{2(\gamma -1)}} \quad . . . (18)$$

Combining equations 14, 15, and 18,

$$\frac{W \sqrt{To_2}}{A_2\, P_1} = \sqrt{\frac{2\gamma g}{R(\gamma -1)}} \sqrt{\frac{1}{\left(\frac{\gamma -1}{2}\right) \frac{\omega^2 r^2{}_2}{\gamma g R T o_2} + \frac{P_3}{P_1}^{\frac{\gamma -1}{\gamma}}} - 1} \left[\frac{1}{\left(\frac{\gamma -1}{2}\right) \frac{\omega^2 r^2{}_2}{\gamma g R T o_2} + \frac{P_3}{P_1}^{\frac{\gamma -1}{\gamma}}}\right]^{-\frac{(\gamma + 1)}{2(\gamma -1)}}$$

$$. . . . . (19)$$

or,

$$\frac{W \sqrt{To_2}}{A_2\, P_1} = \sqrt{\frac{2\gamma g}{R(\gamma -1)}} \sqrt{1 - \left[\frac{P_3}{P_1}^{\frac{\gamma -1}{\gamma}} + \frac{\gamma -1}{2} \frac{\omega^2 r^2{}_2}{\gamma g R T o_2}\right]} \quad x$$

$$\left[\frac{P_3}{P_1}^{\frac{\gamma -1}{\gamma}} + \frac{\gamma -1}{2} \frac{\omega^2 r^2{}_2}{\gamma g R T o_2}\right]^{\frac{1}{\gamma -1}} \quad . . . . (20)$$

The above equation relates weight flow $\left(\frac{W}{A_2}\right)$, pressure drop $\left(\frac{P_3}{P_1}\right)$, turbine tip speed (ωr) and total temperature of the nozzles (To_2). In a conventional axial type turbine only three of the above variables are present, the flow thru the turbine being independent of the speed. Since speed ($r\omega$) is included in the centripetal turbine the overall analysis is complicated by having four variables present instead of the normal three.

Since there are four variables present in equation (20) it was decided to rearrange the variables into parameters each containing two variables so that one family of curves could be plotted. This would allow a graphical solution to the equation when any three variables were fixed.

Equation (20),

$$\frac{W\sqrt{To_2}}{A_2 P_1} = \sqrt{\frac{2\gamma g}{R(\gamma-1)}}\sqrt{1-\left[\frac{P_3}{P_1}^{\frac{\gamma-1}{\gamma}}+\frac{\gamma-1}{2}\frac{\omega^2 r^2}{\gamma gRTo_2}\right]}\left[\frac{P_3}{P_1}^{\frac{\gamma-1}{\gamma}}+\frac{\gamma-1}{2}\frac{\omega^2 r^2}{\gamma gRTo_2}\right]^{\frac{1}{\gamma-1}} \quad \ldots\ldots(21)$$

or,

$$\frac{W\omega r}{A_2P_1} = \sqrt{\frac{2\gamma^2 g^2}{\gamma-1}}\sqrt{\frac{\omega^2 r^2{}_2}{\gamma gRTo_2}}\sqrt{1-\left[\frac{P_3}{P_1}^{\frac{\gamma-1}{\gamma}}+\frac{\gamma-1}{2}\frac{\omega^2 r^2}{\gamma gRTo_2}\right]} \quad x$$

$$\left[\frac{P_3}{P_1}^{\frac{\gamma-1}{\gamma}}+\frac{\gamma-1}{2}\frac{\omega^2 r^2}{\gamma gRTo_2}\right]^{\frac{1}{\gamma-1}} \quad \ldots\ldots(22)$$

Equation (22) can be plotted for given values of $\left(\frac{P_3}{P_1}\right)$ and the results will be a family of curves of $\left(\frac{W\omega r}{A_2P_1}\right)$ vs $\left(\frac{\omega^2 r^2}{\gamma gRTo_2}\right)$. See figure (1) for plot of equation (22).

Appendix C

CENTRIFUGAL COMPRESSOR CHARACTERISTICS

The characteristics of the centrifugal compressor were obtained from reference (2). Values of pressure ratio and $\frac{Q}{ND^3}$ are plotted versus $\frac{\omega^2 r^2}{\gamma gRT}$ for a family of single stage centrifugal compressors. The single stage compressor resulted in a small $\frac{P_2}{P_1}$ and $\frac{Q}{ND^3}$ for the limiting value of tip speed we desired to use, therefore, it was decided to use a double impeller first stage and a single impeller second stage compressor operating at equal values of tip speeds. The characteristic curves of this two stage centrifugal compressor were determined in the following manner.

Since a double flow first stage impeller was used the values of $\frac{Q}{ND^3}$ just doubled the single stage value, when operating at equal tip speeds. The overall pressure ratio of the compressor was taken as the product of the two stages. Since first and second stage impellers were operating at the same tip speed, values of $\left(\frac{P_2}{P_1}\right)$ versus (ωr) could be plotted. Figures (2) and (3) give the resulting values of $\left(\frac{P_2}{P_1}\right)$ and $\left(\frac{Q}{ND^3}\right)$ for tip speeds within the operating range desired. For this family of compressors the approximate diameter of the impeller was (twelve) inches, therefore, the double flow compressor was assumed to have an impeller diameter of (one) foot.

Values of weight flow in lbs. per second for a given tip speed were determined as follows:

For a given tip speed figure (3) gives maximum and minimum values of $\frac{Q}{ND^3}$.

For a given tip speed

$$\text{let } \frac{Q}{ND^3} = Y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$W = \rho AV_g = \frac{P_o Q_o}{R\ T_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Subscript (o) refers to standard conditions of pressure and temperature.

$$N = \frac{\omega}{2\pi} = \frac{\omega r}{2\pi r} \quad \text{R.P.S.} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

From equation (1), (2), and (3)

$$W = \frac{P_o}{RT_o} ND^3\ Y = \frac{P_o}{RT_o} \frac{\omega r}{2\pi r} \quad 8\ r^3\ Y \quad \ldots\ldots\ldots\ldots \quad (4)$$

or,

$$W = \frac{P_o}{RT_o \pi} D^2\ Y\ (\omega\ r) \quad \text{- - -lbs/sec} \quad \ldots\ldots\ldots\ldots \quad (5)$$

Using the following standard conditions

$P_o = 14.7 \times 144$ lbs/ft^2

$R = 53.3$ Air gas constant

$T_o = 70^oF$ to 530^oR

$$\frac{W}{D^2} = .0238\ (\omega\ r)\ Y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$\frac{W}{D^2}$ has maximum and minimum values corresponding to the maximum and minimum values of $\frac{Q}{ND^3}$.

For plot of $\left(\frac{W}{D^2}\right)$ versus $(\omega\ r)$ see figure (4).

For complete compressor data see Table II.

It was assumed that the value of adiabatic efficiency for the theoretical two stage compressor would have the same characteristic curve as the single stage compressor.

The pressure coefficient of a centrifugal compressor is defined as follows:

$$\eta = J g\, c_p \left[\frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\omega^2 r^2} \right] T_1 \quad \ldots\ldots\ldots\ldots\ldots (7)$$

The pressure ratio $\left(\frac{P_2}{P_1}\right)$ is defined as a ratio of the total absolute pressure at the discharge of a compressor to the absolute pressure of the inlet. Values of pressure coefficient at various tip speeds are shown in figure (4) and Table II. Since this parameter remains almost constant within the selected operating range of the compressor, see figure (4), a value of $\eta = 1.74$ was selected for all compressor turbine calculations.

Appendix D

DEVELOPMENT OF COMPRESSOR TURBINE EQUATION

The development of an equation linking the flow conditions of the compressor and centripetal turbine was determined in the following manner.

Since the power output of the turbine is equal to the compressor work, it follows that,

turbine work = compressor work

$$W\ C_{pt}\ T_4\ e_t \left[1 - \frac{P_5}{P_4}^{\frac{k-1}{k}}\right] = \frac{W\ C_{pc}\ T_1}{e_c}\left[\frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} - 1\right] \quad \ldots (1)$$

For constant weight flow,

$$\frac{P_5}{P_4}^{\frac{k-1}{k}} = 1 - \frac{C_{pc}\ T_1 \left[\frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} - 1\right]}{C_{pt}\ T_4\ e_c\ e_t} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

(Refer to sketch (one) for subscripts)

Using the compressor constant called "pressure coefficient" as explained in appendix (C), we have,

$$\eta = J_g\ c_p\ T_1 \frac{\left[\frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} -1\right]}{\omega_c^2\ r_c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Since the pressure coefficient remains constant within the operating range (see figure 4), substituting (3) in (2) we get,

$$\frac{P_5}{P_4}^{\frac{k-1}{k}} = 1 - \frac{C_{pc}}{C_{pt}\ e_c\ e_t}\ \frac{\omega_c^2 r_c^2}{T_4}\ \frac{\eta}{J_g\ C_p} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

$$\text{Let } K_1 = \frac{\omega_c^2 r_c^2}{\omega_t^2 r_t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\text{Let } K_2 = \frac{\eta}{J_g \; C_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(6)$$

K_1 and K_2 are constants within the operating range, therefore, using equations (5) and (6) in (4)

$$\left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}} = 1 - \frac{C_{pc} \; k_g \; R \; K_1 \; K_2}{C_{pt} \; e_c \; e_t} \; \frac{\omega_t^2 \, r_t^2}{k_g \; RT_4} \quad \ldots\ldots\ldots\ldots\ldots.(7)$$

From appendix (B) the turbine equation (22) can be written using present subscripts as:

$$\frac{W \omega r}{A_t P_4} = \sqrt{\frac{2 \; k^2 \; g^2}{k-1}} \sqrt{\frac{\omega^2 r^2}{kgRT_4}} \sqrt{1 - \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}} + \frac{k-1}{2} \; \frac{\omega^2 r^2}{kgRT_4}} \quad x$$

$$\left[\left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}} + \frac{k-1}{2} \; \frac{\omega^2 r^2}{kgRT_4}\right]^{\frac{1}{k-1}} \quad \ldots\ldots(8)$$

Substituting equation (7) in (8)

$$\frac{W \omega r}{A_t \; P_4} = \sqrt{\frac{2k^2g^2}{k-1}} \sqrt{\frac{C_{pc} \; kg \; R \; K_1 \; K_2}{C_{pt} \; e_c \; e_t} - \frac{k-1}{2} \cdot \frac{\omega^2 r^2}{kgRT_4}}$$

$$\left[1 - \frac{\omega^2 r^2}{kgRT_4} \left(\frac{C_{pc} \; kg \; R \; K_1 \; K_2}{C_{pt} \; e_c \; e_t} - \frac{k-1}{2}\right)\right]^{\frac{1}{k-1}} \quad \ldots.(9)$$

Equation (9) relates the compressor and turbine and when plotted as shown in figure (5) it allows a convenient graphical solution to a rather complex equation.

The following values were used in determining the constant:

$C_{pc} = 0.241$
$C_{pt} = 0.271$
$e_c = .80$
$e_t = .85$
$K_1 = .720 = \left(\frac{1020}{1200}\right)^2$
$K_2 = 2.88 \times 10^{-4}$
$K = 1.34$

Using the above constants, equation (9) reduced to,

$$\frac{W \omega r}{A_t P_4} = 70.2 \frac{\omega^2 r^2}{kgRT_4} \left[1 - 0.453 \frac{\omega^2 r^2}{kgRT_4}\right]^{\frac{1}{k-1}} \quad \ldots \ldots \ldots (10)$$

(See figure (5) for plot).

Appendix E

TURBINE CHARACTERISTICS TO MEET MARINE POWER REQUIREMENTS

With development of the gas generator unit of the power output for any given condition of the first unit could be found. The work in this appendix is an attempt to fit this power output to the speed HP requirements of a typical marine installation, assuming the power fitted the ideal marine requirement. $HP = KN^3$.

The following assumptions were made:

1. HP of the ship was directly proportional to the shaft speed cubed.
2. Centripetal turbine efficiency was constant throughout the load range.
3. C_p was constant (averaged thru the temperature range).
4. The nozzle temperature could be held constant at its maximum value.

At maximum load conditions,

$$HP_o = KN_o^3 \qquad \text{. (1)}$$

At any condition,

$$HP_x = KN_x^3 \qquad \text{. (2)}$$

The ratio of (1) and (2),

$$\frac{HP_x}{HP_o} = \frac{KN_x^3}{KN_o^3} \qquad \text{. (3)}$$

$$N = \frac{\omega}{2\pi} = \frac{r_t \omega t}{2\pi r_t} \qquad \text{. . . . . (4)}$$

If N is RPM and is per second,

$$N = \frac{\omega_t r_t}{2 \ r_t} \ (60) \text{ second} \quad \dots\dots\dots\dots\dots (5)$$

Equation (5) in (3),

$$\frac{HP_x}{HP_o} = \frac{\omega r_x{}^3}{\omega r_o{}^3} \quad \dots\dots\dots\dots\dots\dots\dots (6)$$

$$HP_o = W_o \ C_p \ e_t \ T_6 \left(1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right)_o \quad \dots\dots\dots\dots (7)$$

Combining (6) and (7),

$$\frac{\omega r_x{}^3}{\omega r_o{}^3} = \frac{W_x \ C_p \ e_t \ T_6 \left(1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right)_x}{W_o \ C_p \ e_t \ T_6 \left(1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right)_o} \quad \dots\dots\dots (8)$$

$$\omega r_x{}^3 = \frac{W_x}{W_o} \ \frac{\left(1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right)_x}{\left(1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right)_o} \ (\omega r_o)^3 \quad \dots\dots\dots (9)$$

Equation (9) shows the relationship which must be met if the power turbine is to vary its load and speed in the same manner as the marine requirements.

Appendix F

METHOD OF CYCLE CALCULATION

The preceeding appendices have developed the centripetal turbine theory and methods of solving the turbine equation. From the various curves plotted for the turbine the following trial and error method of cycle calculation was evolved.

The following cycle conditions were assumed.

1. Maximum cycle temperature ... 1500°F
2. Maximum turbine tip speed ... 1200 ft/sec.
3. Maximum compressor tip speed. 1020 ft/sec.
4. Adiabatic compressor efficiency 80%
5. Adiabatic turbine efficiency .. 85%
6. C_p of compressor 241
7. C_p of burner 260
8. C_p of turbine 271
9. γ of compressor............1.395
10. k of turbine 1.34
11. f for regenerator.......... 75%

In the following analysis the cycle will be designed for maximum load conditions and a so called minimum load point will be chosen to indicate the method of solution. Intermediate points have been calculated and the results are shown in table VI. For cycle arrangement refer to sketch (one).

Compressor Turbine Unit.

Refering to figure (4) the maximum weight flow selected for the compressor was 13.0 lbs/sec. This value is well within the compressor stability limits. Likewise the minimum weight flow was selected at 6.0 lbs/sec. The selection of the weight flows at a given compressor speed will effect the turbine nozzle area and nozzle temperature, therefore, the selection is up to the designer. Resulting calculations will prove whether the above selection was justified.

Turbine No. 1 Maximum conditions:

$$W = 13.0 \text{ lbs/sec.}$$

$$T_4 = 1960^{\circ}R$$

$$\omega r = 1200 \text{ ft/sec.}$$

$$\therefore \frac{\omega^2 r^2}{kgRT_4} = 0.320 \quad \ldots\ldots\ldots\ldots (1)$$

Entering figure (5) obtain the values

$$\frac{W \omega r}{A_t \; P_4} = 14.20 \quad \ldots\ldots\ldots\ldots (2)$$

$$\frac{P_4}{P_5} = 2.39 \quad \ldots\ldots\ldots\ldots (3)$$

Checking for acoustic choke figure (7) we are just within the limiting value of pressure ratio. From appendix (D)

$\omega_c \, r_c = \sqrt{K_1} \; \omega_t r_t$ and $\therefore \omega_c r_c = \sqrt{.720} \times 1200 = 1019$ ft/sec.

From figure (2),

$$\frac{P_2}{P_1} = 4.90 \quad \ldots\ldots\ldots\ldots (4)$$

Assuming no losses between the compressor and turbine, the residual pressure ratio is,

$$\frac{P_6}{P_7} = \frac{P_2}{P_1} \times \frac{P_5}{P_4} = 2.05 \quad \ldots\ldots\ldots\ldots (5)$$

The turbine nozzle area can be calculated from (2)

$$A_t = \frac{W\,\omega r}{P_4 \times 14.20} = 15.25 \text{ sq. inches} \ldots\ldots\ldots (6)$$

Proceeding as above for the minimum conditions using the calculated nozzle area;

Minimum Conditions

$W = 6.0$ lbs/sec.

$\omega r_t = 800$ ft/sec.

$\omega r_c = 679$ ft/sec.

$\frac{P_2}{P_1} = 2.15$ from figure (2),

Calculate $$\frac{W\,\omega r}{A_t\,P_4} = \frac{6 \times 800}{15.25 \times 2.15 \times 14.7} = 9.95 \ldots\ldots (7)$$

Using figure (5),

$$\frac{\omega^2 r^2}{kgRT_4} = .1820 \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\frac{P_4}{P_5} = 1.61 \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

From (8) $T_4 = 1529°R \ldots\ldots\ldots\ldots\ldots\ldots (10)$

$$\frac{P_6}{P_7} = \frac{2.15}{1.61} = 1.335 \ldots\ldots\ldots\ldots\ldots (11)$$

Proceeding as above making judicious selections of weight flows at various tip speeds a reasonable temperature schedule for turbine number (1) was calculated. This located the gas generator unit in the cycle, the residual pressure, and weight flow schedule to turbine number (2).

The conditions of flow to turbine number (2) are:

	Weight Flow W	Pressure Ratio P_6/P_7
Maximum:	13.0	2.05
Minimum:	6.0	1.335

Computing the area of turbine number (2) for the maximum conditions:

$$W = 13.0 \quad \text{lbs/sec.}$$
$$\omega r = 1200 \quad \text{ft/sec.}$$
$$\frac{P_6}{P_7} = 2.05$$
$$T_6 = 1960^{\circ}R$$

$$\therefore \frac{\omega^2 r^2}{kgRT_6} = .320 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

From figure (1) (turbine equation) for a pressure ratio of 2.05

$$\frac{W\omega r}{A_t\ P_6} = 13.90 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Since W, ωr, and P_6 are known the nozzle area of turbine number (2) is,

$$A_t = \frac{W\omega r}{13.90 \times P_6} = 37.20 \text{ sq. inches} \quad \ldots\ldots (14)$$

Minimum Conditions Turbine Number (2).

Since the power output of this cycle must satisfy marine power characteristics we have the following relationship between W, ωr, and P_7/P_6 from appendix (E).

$$(\omega r_t)_x^3 = \frac{W_x}{W_o} (\omega r)_o^3 \quad \frac{\left[1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right]_x}{\left[1 - \frac{P_7}{P_6}^{\frac{k-1}{k}}\right]_o} \quad \ldots\ldots\ldots (15)$$

Subscript (x) refers to any part load condition.

Subscript (o) refers to the maximum condition.

Substituting the proper values in (15)

$$\omega r \text{ (min)} = 697 \quad \text{ft/sec.}$$

$$\therefore \quad \frac{\omega^2 r^2}{kgRT_6} = .1071$$

Using figure (1) for pressure ratio of 1.335,

$$\frac{W \omega r}{P_6 A_t} = 6.69 \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

We can calculate $\frac{W \omega r}{P_6 A_t}$ for designed area,

$$\therefore \frac{W \omega r}{P_6 A_t} = 5.73 \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Comparing the two values of $\frac{W \omega r}{A_t P_6}$ it can be seen that they are not in agreement. This means that our selection of variables for turbine number (2) are incorrect. For the conditions of tip speed and temperature assumed the curve gives the correct value of the above parameter. Since it was decided to maintain T_6 constant we could either change the weight flow or pressure ratio to the turbine so that the power requirements and turbine requirements would be satisfied simultaneously. It was decided to change the weight flow and keep the pressure ratio as scheduled from turbine number (1). This involved a trial and error solution to satisfy the conditions of turbine number (2). Also, changing the weight flow schedule required another trial and error solution for turbine number (1), in determining its new speed and temperature schedule to satisfy the revised weight flow.

Increasing W to 6.92 lbs/sec. gives a new speed of,

$\omega r = 729$ ft/sec.

$\frac{P_6}{P_7} = 1.335$ (this value was not changed)

From figure (1) $\frac{W \omega r}{A_t\ P_6} = 6.92$ . . . .(18)

Calculated $\frac{W \omega r}{A_t\ P_6} = 6.92$ . . . . (19)

All the variables of turbine number (2) are now satisfied so returning to turbine number (1) with the new weight flow and same pressure ratio schedule, these conditions must be satisfied by changing the speed and consequently the temperature schedule of turbine number (1).

The maximum designed condition of turbine number (1) will not change, since no change was made in the maximum weight flow and residual pressure ratio, therefore, the nozzle area of turbine number (1) is still 15.25 square inches. However, the minimum and all other selected points have been altered. In order to satisfy all conditions we must resort to another trial and error solution.

Assume $\omega r_t = 850$ ft/sec.

$\therefore$ $\omega r_c = 722$ ft/sec.

$\frac{P_2}{P_1} = 2.39$ figure (2)

Calculate $\frac{W \omega r}{A_t\ P_4} = 10.98$ . . . .(20)

From figure (5) $\frac{P_4}{P_5} = 1.730$ . . . .(21)

Residual Pressure $\frac{P_6}{P_7} = 1.370$ . . . .(22)

Since the above $\left(\frac{P_6}{P_7}\right)$ is not the correct value that was obtained previously, by trial and error the correct speed can be found to satisfy our original conditions.

Assume $\omega r_t = 840$ ft/sec.

$\omega r_c = 714$ ft/sec.

$P_2/P_1 = 2.33$

Calculate $\frac{W \omega r}{A_t\ P_4} = 11.10$ . . . .(23)

$P_4/P_5 = 1.745$ figure (5) . . . .(24)

$\frac{\omega^2 r^2}{kgRT_4} = .212$ figure (5) . . . .(25)

Calculate $\frac{P_6}{P_7} = 1.335$ This checks with original pressure ratio.

From (25) $T_4 = 1448^oR$ . . . . (26)

The final results of the previous calculations are:

Cond.	W #/sec	T_4 ^oR	T_6 ^oR	Turbine #1 ωr_t	ωr_c	Turbine #2 ωr	$\frac{P_2}{P_1}$	$\frac{P_4}{P_5}$	$\frac{P_6}{P_7}$
Max.	13.0	1960	1960	1200	1019	1200	4.9	2.39	2.05
Min.	6.92	1448	1960	840	714	729	2.33	1.745	1.335

The cycle operating on the temperature schedule obtained in the above manner forms a stable unit fulfilling the marine output requirements.

For the complete data for intermediate points refer to table VI.

Temperature Calculations:

$$T_2 = T_1 \left[\frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{e_c} + 1 \right] = \begin{array}{l} 907^\circ R \text{ Maximum} \\ 709^\circ R \text{ Minimum} \end{array}$$

$$T_3 = f \left[T_7 - T_2 \right] + T_2 = \begin{array}{l} 1490^\circ R \text{ Maximum} \\ 1559^\circ R \text{ Minimum} \end{array}$$

$$T_5 = T_4 \left[1 - e_t \left(1 - \frac{P_5}{P_4}^{\frac{k-1}{k}} \right) \right] = \begin{array}{l} 1630^\circ R \text{ Maximum} \\ 1285^\circ R \text{ Minimum} \end{array}$$

$$T_7 = T_6 \left[1 - e_t \left(1 - \frac{P_7}{P_6}^{\frac{k-1}{k}} \right) \right] = \begin{array}{l} 1683^\circ R \text{ Maximum} \\ 1844^\circ R \text{ Minimum} \end{array}$$

Efficiency Calculations:

$$\text{Efficiency} = \frac{\text{turbine output}}{\text{fuel input}} =$$

$$\text{Efficiency} = \frac{W\, C_{pt} \qquad T_6 - T_7}{W\, C_{pb} \quad (T_4 - T_3) + (T_6 - T_5)} = \begin{array}{l} 36.0\% \text{ Maximum} \\ 18.0\% \text{ Minimum} \end{array}$$

Refer to table VI for complete data. Plotted data is shown in figures 8, 9, 10, and 11.

The introduction of a regenerator with an effectiveness of .75%, produces a limitation on T_4 at the minimum loads. To remedy this an exhaust dumping system could be provided so that just enough gases would be dumped to allow T_3 to approach the scheduled values of T_4. For lower loads burner number (1) could be shut off and temperature regulated by the gas dumping mechanism. Lower loads could also be achieved by decreasing temperature to turbine number (2).

DATE DUE

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